LAB REPORT

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***Abstract*— control the behaviour or the functionality of some devices is important for engineering, developing new techniques and methods is always required, and the use of loops can help engineers to develop devices, implementation of open loops and closed loops in devices, help achieve the desired output response, more methods to achieve more accurate devices can be the implementation of PID controllers, during the study of different tests where some specific characteristics will be taken to obtain the transfer function of the system and simulate this response using software applications such as Simulink, it possible to understand how these methods affect the systems in terms of an output response, providing a good methodology that will help to configure systems according to a specific desired output.**

***Index Terms*—time constant, Simulink, PID**

I. INTRODUCTION

The purpose of this report is to analyse the differences between two of the more common control systems, through the realisation of various tests, the reader would understand the advantages and disadvantages of each control system, also the implementation of a Proportional-Integral-derivative (*PID*) controllers used in closed loops, and finally an explanation of how to recreate or represent these control systems responses using software applications such as Simulink.

TABLE I

Open Loop System	**Closed Loop System**
Accuracy is low	High accuracy
Not reliable	More reliable
Easy to design	Complex in design
Less expensive	Expensive
No feedback	Feedback

II. THEORY

A. Open Loop

An open loop Control System is a system in which the control action is independent of the output of the system, in other words, there is no feedback sent to the system to inform if is required to increase or decrease the input to achieve the desired output, this means that the accuracy of the system will depend on the user

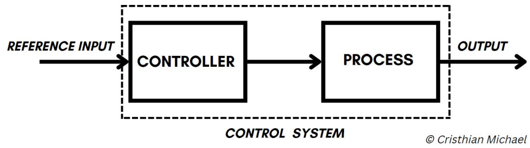

Figure 1: Open Loop System

Some of the advantages of this system are the simplicity of the construction, which helps to reduce the cost of this type of system, however, some important disadvantages are that this system is not equipped to handle disturbance, giving a wrong output caused by the disturbance, this means that Open Loops are not reliable.

To calculate the output response of an Open Loop System, the blocks must be combined to obtain the open loop's transfer function.

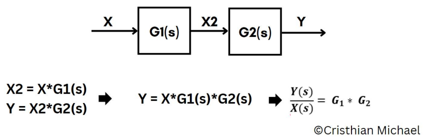

Figure 2: Open Loop Transfer Function

B. Closed Loop

A closed-loop system follows the same control systems as the one used in an open loop, however, this system includes feedback that consists on measured the output continuously, and this data is sent back to the input, this means that the input will change to achieve the desired output, another difference of the closed-loop system with the open loop system is that the closed loop can handle the disturbance caused by external factor through the use of the feedback that compensates the disturbance improving the accuracy of the system.

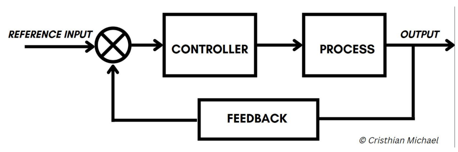

Figure 3: Closed Loop System

To obtain the transfer function of a closed-loop system is required to analyze new signals that are not incorporated in the open loop, for example, the error signal and the feedback signal, these new signals will adjust the input to obtain the desired output, the final equation can vary depending on if is a negative or positive feedback.

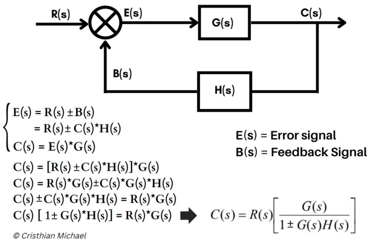

Figure 4: Closed Loop Transfer Function

C. PID Controller

To improve the performance of a closed-loop system, a PID controller can be integrated into the system, this controller has three key components that will help to achieve the desired output response, the first component is the proportional gain (K_p), the output respond obtained from this control, depends on the gain and the error of the system, this error comes from the subtraction of the desired or set point (*SP*) output with the current output also called process variable (*PV*)

$$error = SP - PV$$

Equation 1: Error calculation.

This error is multiplied by the gain in order to achieve the desired output.

$$Output\ signal = Error * K_p$$

Equation 2: Output calculations using the K_p

The second controller is the integral gain (K_i) the use of this controller can help to reduce one problem that the proportional gain controller cannot avoid, and it is that the error cannot be zero, so the steady error can be decreased but not eliminated, however with this new controller the steady-state error can be shown with an error of zero, but this is not always a reality because the system may be oscillating, however, this oscillations can be so small that the error seems to be zero. (Peterson, 2020)

$$Output\ signal = K_i * \int_0^t Error * (\tau) d\tau$$

Equation 3: Integral calculation.

The last controller is called derivative gain K_d , the function of this controller is to decrease the overshoot and settling time of the output response of the system by multiplying the derivative gain by the differentiation of the error in the system.

$$Output\ Signal = K_d * \frac{de\,(t)}{dt}$$

Equation 4: Differentiation calculation

PID controllers are very useful to solve a wide range of control problems where precision and accuracy are required, a large number of control problems in industries have been solved using these components, because of their sophisticated methods. (Aström, 2009)

TABLE II

	K_p	K_i	K_d
P	$0.5K_{ult}$	0	0
PI	$0.45K_{ult}$	$0.83P_{ult}$	0
PID	$0.6K_{ult}$	$0.5P_{ult}$	$0.125P_{ult}$

III. RESULT

A. Open Loop

Using an Arduino to control a motor ring, it is possible to extract the transfer function of the system, by observing the graph that will be displayed, once a step input is applied, using the data provided by the graph, the standard transfer function of a first-order system can use.

$$G(s) = \frac{k}{s + a}$$

Equation 5: Standard transfer function

This equation can be converted into a unity constant form.

$$G(s) = \frac{\frac{k}{a}}{\frac{s}{a} + 1}$$

Equation 6: Unity constant form.

1) Test 1

Setting a set point to 1V, it is possible to observe that the maximum value of revolution per minute (*RPM*) is approximately 3315 RPM, to obtain the transfer function of the open loop is required to calculate the 63.2% of the maximum value of RPM, obtaining a value of 2095 RPM, using this value and the graph it is possible to obtain the time constant that in this case is 0.53 seconds

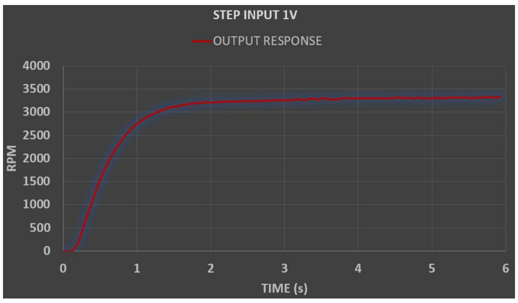

Figure 5: Step Input 1V

The transfer function obtained is in the standard form so it is possible to observe the time constant and the maximum value that the output will reach.

$$G(s) = \frac{3315}{0.53s + 1}$$

Equation 7: First order Transfer Function.

Another way of representing the first-order transfer function of the open loop is by using the standard form, to obtain this form, the process followed was to use the unity form equation, and find the value *a*, by using the time constant and the maximum value that the output response reaches,

$$G(s) = \frac{\frac{k}{a}}{\frac{s}{a}+1} \rightarrow \frac{\frac{6254}{1.886}}{\frac{s}{1.886}+1} \rightarrow \frac{6254}{s+1.886}$$

$$\frac{1}{a} = 0.53 \rightarrow \frac{1}{0.53} = a \rightarrow 1.886 = a$$

$$\frac{k}{a} = 3315 \rightarrow \frac{k}{1.886} = 3315 \rightarrow k = 6254$$

Figure 6: Workout calculations

The equation found, is the standard form of the transfer function obtained previously.

$$\frac{6254}{s + 1.886}$$

Equation 8: Standard form

The transfer function can rescale it in terms of RPM, to do this the numerator has to be divided by the maximum value that reaches the output response, therefore the equation in terms of RPM would be the following.

$$\frac{1.886}{s + 1.886}$$

Equation 9: RPM form

With the transfer function obtained, it is possible to represent the output response of the motor at 1V, by using Simulink.

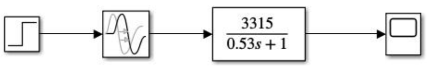

Figure 7: Simulink Step Input 1V

Once the block diagram has been created in Simulink, the code can be run obtaining a graph similar that the one obtained from the test.

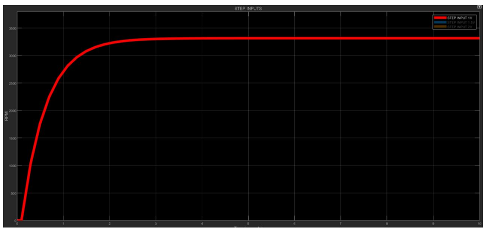

Figure 8: Simulink simulation 1V step input

2) Test 2

Setting a set point to 1.5V, it is possible to observe that the maximum value of RPM is approximately 6435 RPM, to obtain the transfer function of the open loop is required to calculate the 63.2% of the maximum value of RPM, obtaining a value of 4066 RPM, using this value and the graph it is possible to obtain the time constant that in this case is 0.49 seconds

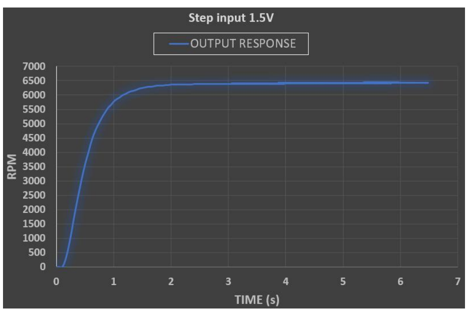

Figure 9: Step Input 1.5V.

The following equation is obtained by placing these values into the transfer function for a first-order system.

$$G(s) = \frac{6435}{0.49s + 1}$$

Equation 10: First Order Transfer Function

Using Simulink and the equation obtained, it is possible to recreate the output response of the motor with a step input of 1.5V, by using Simulink.

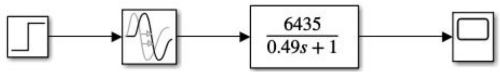

Figure 10: Simulink Step Input 1.5V

Once the block diagram has been created, it is possible to observe that the graph obtained is similar that the one obtained from the test of the step input of 1.5V.

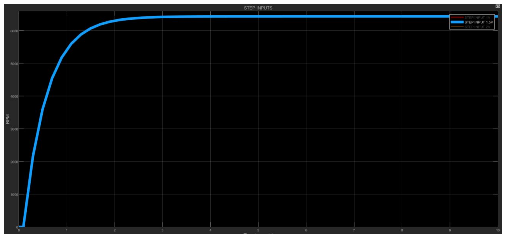

Figure 11: Simulink simulation 1.5V step input.

3) Test 3

Setting a set point to 2V, it is possible to observe that the maximum value of RPM is approximately 9496 RPM, to obtain the transfer function of the open loop is required to calculate the 63.2% of the maximum value of RPM, obtaining a value of 6001 RPM, using this value and the graph it is possible to obtain the time constant that in this case is 0.49 seconds

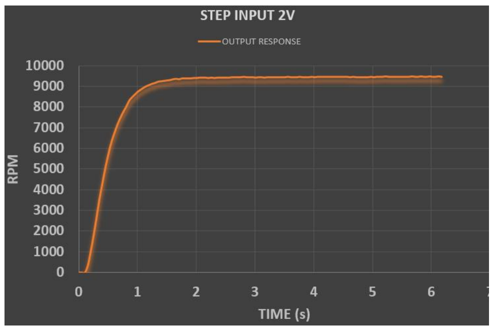

Figure 12: Step Input 2V

The following transfer function will be used to recreate the output response of the motor when the step input is 2V.

$$G(s) = \frac{9496}{0.49s + 1}$$

Equation 11: First Order Transfer Function

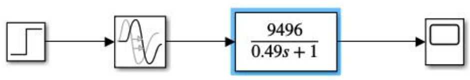

Figure 13: Simulink connection

Using the block diagrams, the output response of the motor was obtained, getting a similar response that the one obtained from the test.

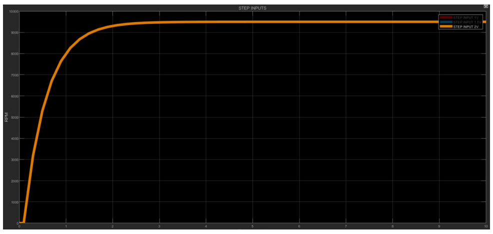

Figure 14: Simulink simulation 2V step input.

4) Combination

In order to study the transfer functions for each step input, the three graphs obtained previously have been plotted in a single graph.

TABLE III

Step Input	**1V**	**1.5V**	**2V**
Time Constant	**0.53s**	**0.49s**	**0.49s**
Max value	**3315 RPM**	**6435 RPM**	**9496 RPM**
Function	$\frac{3315}{0.53s+1}$	$\frac{6435}{0.49s+1}$	$\frac{9496}{0.49s+1}$

It is possible to observe that there is a correlation between the step input and the maximum value of RPM, as the step input increase, the maximum value of RPM increases.

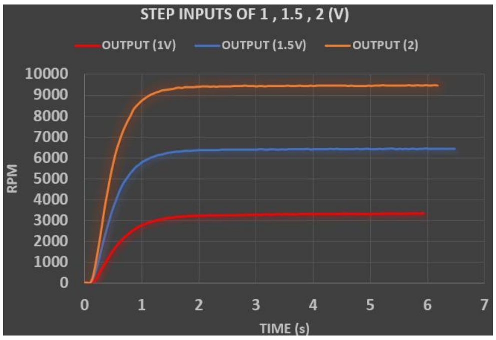

Figure 15: Comparison of step inputs.

Using Simulink and the transfer function for each step input, it is possible to plot the three transfer functions, obtaining a similar result that the ones obtained from Excel, this verifies that the equations obtained are reliable.

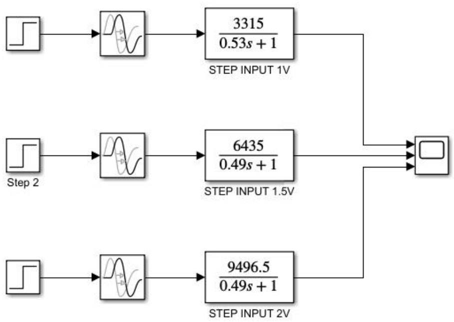

Figure 16: Simulink schematic representation.

Observing the three graphs, it is possible to identify that the time constant for each transfer function is almost the same.

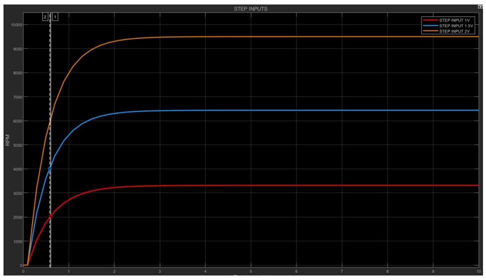

Figure 17: Simulink step input comparison.

5) Disadvantage

One of the most important characteristics of an open-loop system is that it cannot compensate for any disturbance for example, if an external factor reduces the speed of the motor, the output response will be corrupted, to prove this, the Arduino and the motor were connected to a DC power supply, with a step input of 1V, the motor was gently touched, causing a reduction on the speed, and to make it speed up again it was manually adjusted.

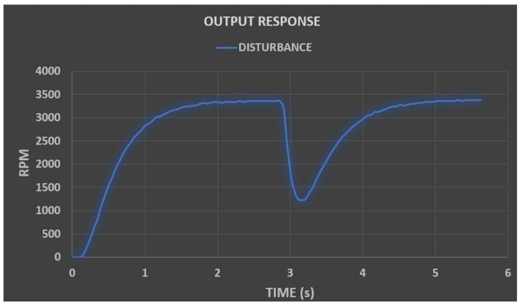

Figure 18: Disturbance in Open Loop.

This experiment proved that open loop systems do not correct disturbance and are simply commanded by the input, the lack of feedback causes the systems not to take samples of the output to recognize if the desired output has been reached and depending on that adjust the system and reduce the error on it. (Nise, 2010)

B. Closed Loop

Using the same motor used for the open loop now is going to be used for the closed loop, one of the characteristics of this control system is that incorporates feedback and a PID controller, which will cause a different output response than the ones observed before.

1) Test 1

Setting the set point to 10000 RPM and the proportional gain (K_p) of 1, the output response of the motor will reach a value of 4720 RPM, this value does not reach the desired response, this is because the K_p used was small, and to reduce the steady error, a higher K_p should be used, this proves that K_p is not the best in terms of reducing the steady-state error.

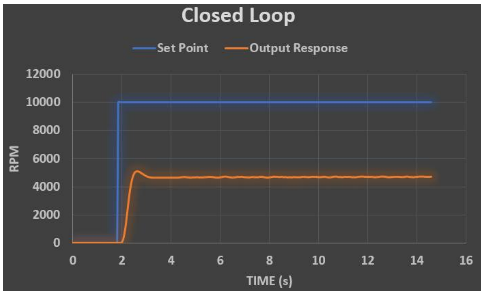

Figure 19: Closed Loop PID

2) *Test 2*

To verify that a high proportional gain is required to reduce the steady-state error, the K_p values have been increased manually until it the output response decreases its oscillation, to achieve this result the value used for the proportional gain was twenty.

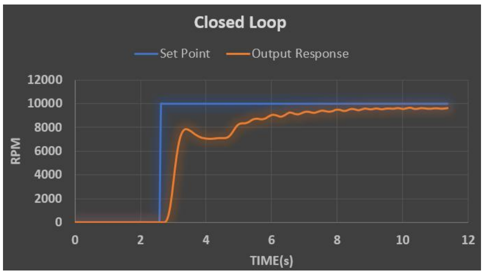

Figure 20: Multiple Kp used

To observe the results in a better way, four $K_p$with values of one, two, four and ten have been set up, by observation, it is possible to find a relation between the amplitude and the proportional gain, when the K_p value increase, the steady state error decrease and the amplitude of the output response increase, however, one disadvantage of this is that the overshoot increase.

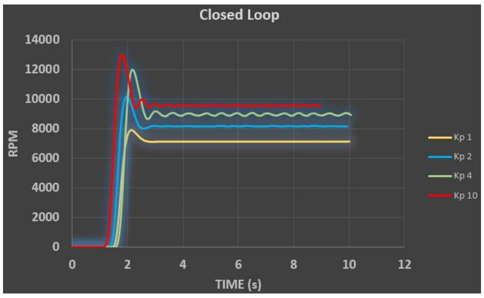

Figure 21: Different Kp used

3) *Test 3*

Using another component of PID controllers called integral gain (K_i), it would be possible to reduce the steady-state error of the output response of the system, setting a K_i of 1 and a K_p of 1, from observation and comparing this result with the one obtained in test 1 where K_p has the same value but K_i is not involved, it is possible to observe how integral gain reduce the steady state error of the system.

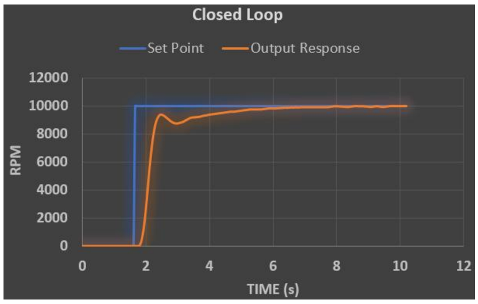

Figure 22: Output response

4) *Test 4*

For this new test, a new K_p of 10 has been set up, with a K_i of 1, comparing this result with the obtained in the previous test, and the one obtained in test 2, where different K_p were studied, it is possible to observe how this new output respond is a combination of those graphs, where the overshoot has increased, and the steady state error has been reduced,

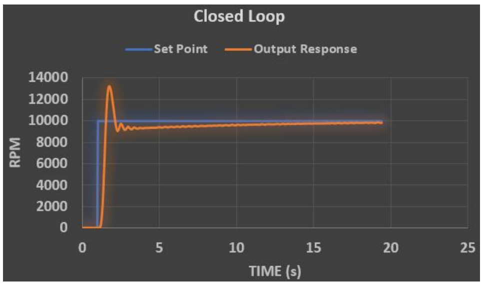

Figure 23: Output response

5) *Test 5*

For this test K_p had a value of 1 and K_i had a value of 10, and observing the output response, it was possible to observe that K_i increases the overshoot like the product gain does, one more thing can be observed from this test and it is that the setting time has increased in comparison from the previous test.

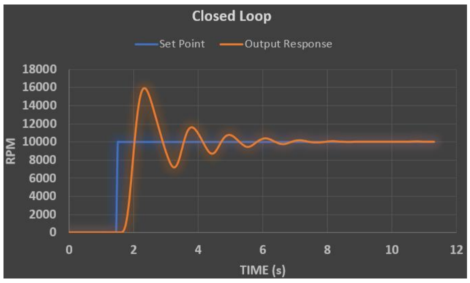

Figure 24: Output response

To verify these variations in the output response, a new value for the integral gain of 30 had been applied, the new response obtained verifies that if the integral gain increase, the settling time increase as well due to an increment of the oscillation on the output response

Figure 25: Output response

6) Test 6

Using the same values for K_p and K_i used in test 5, but incorporating a new component called derivative gain (K_d) with a value of 0.1, the output response of the system seems the same, however, the overshooting has been decreased.

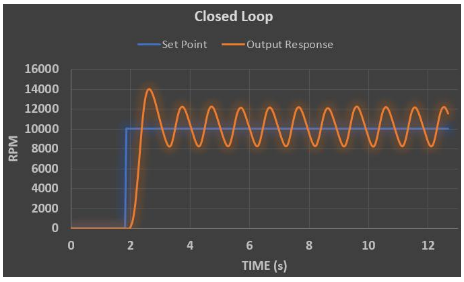

Figure 26: Output response

7) Test 7

Now that the three components that form a PID have been analyzed, the next test was focused on one of the most characteristics that difference an open loop from a closed loop, this characteristic is the correction that the closed loop does when a disturbance affects the output response, setting a value for K_p of 1, and a value for K_i of 10, the motor start to spin and once the output response stable, the motor was touched gently with a card.

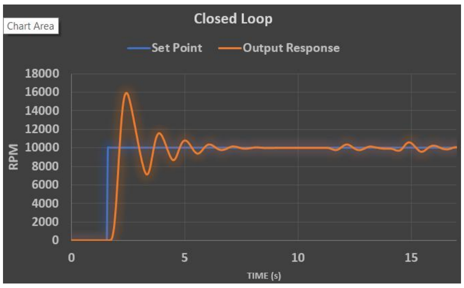

Figure 27: Output response

Observing the output response of the closed loop system, it is possible to observe the disturbance caused by the card, however comparing this response with the one obtained in from the open loop system, it is possible to see how the closed loop can control the disturbance caused by the card automatically, adjusting the speed to reach the desired output respond, on the other hand, the open loop was not able to control the disturbance by itself, and the user has to manually adjust the motor after being affected by the disturbance.

8) Test 8

Using PID controllers to optimize the output response was possible to obtain a fast response with minimal overshoot and a fast-settling time, to achieve this response different PID values were used depending on the response desired.

TABLE IV

PID controller	Function
K_P	Reduce rise time
K_I	Reduce steady error
K_D	Reduce overshoot

The values selected were chosen depending on the effect that each component in the PID controller produces in the output response, so to achieve a fast response, minimal overshoot and fast settling response the following values were used.

TABLE V

K_P	K_I	K_D
3	**5**	**0.51**

With these values, it was possible to observe a fast response, and a minimal overshot, and the steady-state error was almost eradicated.

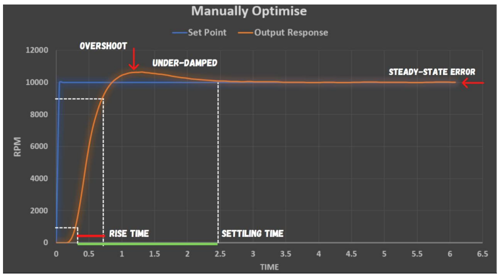

Figure 28: Output response

Another thing that is possible to observe and calculate is the damping in the output response, to obtain the value of the damping the first thing that must be calculated is the magnitude of the first overshoot obtaining a value of 0.064, once this value is obtained it possible to obtain the damping.

$$M_P = \frac{overshoot - setpoint}{Set\ Point} ; \frac{10640 - 10000}{10000} = 64m$$

Equation 12:Magnitude calculation.

Using the formula for the damping ratio, a value of 0.658 was obtained, this verifies that the output response is under-damped because the value obtained was less than 1, there is one more thing to observe from this test, and it is that the overshoot in the output response increase when the damping value decrease

$$\frac{1}{\sqrt{1+\frac{\pi^2}{lnMp^2}}} = \xi ; \frac{1}{\sqrt{1+\frac{\pi^2}{ln0.064^2}}} = 0.658$$

Equation 13: Damping calculation.

Another important characteristic in this graph is the settling time, using the graph and the time is taken to the output response to achieve an equilibrium, in this case, the settling time is around 2.3 seconds, another value important is the time peak, this value is the time that takes to reach the maximum value in the output response, in this case the time peak is 1.1 seconds, once the time peak has been found it is possible to obtain the natural frequency that has a value of 3.8 radians per seconds.

$$w_n = \frac{\pi}{t_p\sqrt{1-\xi^2}}\ ;\ \frac{\pi}{1.1\sqrt{1-0.658^2}} = 3.8$$

Equation 14: Natural frequency calculation

With all these values it is possible to obtain the second-order order transfer function of the system.

$$\frac{3.8^2}{s^2 + 2 * 3.8 * 0.658 * s + 3.8^2}\ ;\ \frac{14.44}{s^2 + 5s + 14.44}$$

Equation 15: Second-order transfer function

9) *Test 9*

For this test, the Ziegler and Nichols table has been used to achieve a reduction in the second overshoot obtaining a value of only 25% of its initial overshoot.

Three values are required for this table (K_p, T_d, T_i) the proportional gain, in this case, would be the maximum value obtained in test two with proportional gain equivalent to twenty, and the differential and integral term, the value of these terms can be observed in the oscillation of the output response, because is the period or time that take the oscillation to complete one cycle when the output response has reached a steady final value, and for this case, that value is 0.3 seconds

TABLE VI

	K_p	T_i	T_d
P	0.5 * 20 = **10**	0	0
PI	0.45 * 20 = **9**	0.83 * 0.3 = **0.25**	0
PID	0.6 * 20 = **12**	0.5 * 0.3 = **0.15**	0.125*0.3=**0.038**

Once the table has been completed, it was possible to use the PID controller with the values obtained to obtain a fast output response, with an overshoot that will decrease to 25%.

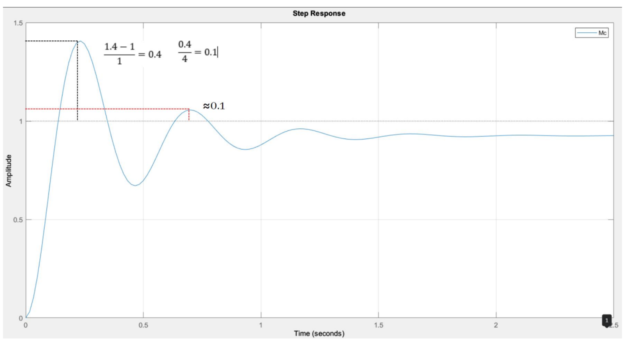

Figure 29: Output response

To prove that the values used in the table were correct, the maximum value of the response was obtained and subtracting the desired input an amplitude of 0.4 was found, this means that the second overshoot should have an amplitude of 0.1, observing the graph it is possible to observe that the second overshoot it almost one, which means that the values used have a small error, but the method is reliable.

IV. CONCLUSION

Depending on the output desired, accuracy and other factors such as the complexity of the design, the use of an open loop or closed loop will be more effective, from the labs was possible to understand the simplicity of an open loop, however its low accuracy is a huge problem if a design require a high accuracy control system, however, the uses of closed-loop provides a better accuracy, but the complexity of the design make them more difficult to design the use of a feedback makes this configuration more reliable because help to reduces the disturbances caused by external sources, and the implementation of a PID controller makes them more manipulable, so the user can obtain the desired output, adjusting the value of the PID controller, so if a design requires a high accuracy, closed loops are the best choice, and if and specific output response is required the implementation of a PID would be beneficial.

V. REFERENCES

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