# Electrical Circuits Assignment (*December 2022*)

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***Abstract*—**There are several ways to analyse circuits, depending on the number of sources, components, or loops, one method can be more appropriate than others, this report consists of the study of the different circuits through the use of hand calculation, analysis of real recreation of those circuits and compares the result obtained between them.

***Index Terms*—** Thevenin's Theorem, KVL, Ohm's Law, RC.

## I. INTRODUCTION

The purpose of this report is to explain and compare, the different theorems and methods to analyse circuits, to achieve this goal different laboratories have been carrying out, where these theorems have been analysed using software applications such as Proteus, hand calculations, and a recreation of the circuits have been done using resistors and multimeters to obtain the actual voltages, to analyse the differences between the results obtained.

## II. THEORY

### A. *Thevenin's Theorem.*

One of the most popular theorems to solve and analyse complex circuits is named after M.L. Thevenin, a French engineer, this theorem consists in simplify large circuits by replacing everything except the load resistance, creating a new circuit called *Thevenin equivalent,* which includes an independent voltage source, the load resistor in series with another resistor. (Hayt, 1993)

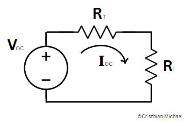


*Figure 1: Thevenin's Equivalent circuit.*

To obtain Thevenin's equivalent, the circuit must be separated into two networks A and B. Network A will contain the complex circuit that will be simplified, and network B will contain the load resistor.

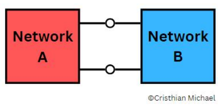


*Figure 2: Networks connection.*

Three parameters need to be calculated to obtain the equivalent circuit, these parameters are *Voltage Thevenin's*, the *short circuit current*, and *Thevenin's Resistance*, consider the Thevenin equivalent with an open-circuited terminal, no current will flow through an open-circuited, which means that no current will flow through the load resistance giving a zero voltage across it, it's possible to apply Kirchhoff's Voltage Law (*KVL*) obtaining that the voltage of the open circuit will be equal to the Thevenin's voltage. (Hambley, 2011)

$$V_{oc} = V_t$$

*Equation 1: Voltage Thevenin's.*

To obtain Thevenin's resistance, it's required to remove all the voltage or current sources, for example, if there is a voltage source, this one must be removed, creating a short-circuit, and if there is a current source, this one must be removed, creating an open circuit, then it's required to obtain the total resistance of the circuit.

$$R_{eq} = R_t$$

*Equation 2: Thevenin's resistance.*

Once all the other parameters have been calculated, it's possible to obtain the current, and because the open circuit and Thevenin's equivalent have the same current, it is possible to use Ohm's Law to find the open circuit current.

$$I_{oc} = \frac{V_t}{R_t}$$

*Equation 3: Current in the circuit.*

### B. *Nodal Analysis and Superposition*

Nodal analysis is a method that allows studying more complex circuits, by analysing the nodes, each node is connected to two or more components in the circuit, and the current will flow through each component passing the node, one of the conditions for Nodal analysis is that the sum of current that is leaving the node must be equal to the sum of current entering in the node.

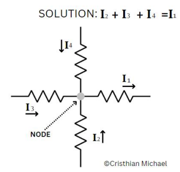


*Figure 3: Nodal Analysis.*

Superposition theorem helps to analyse circuits with more than one voltage or current source, the theorem consists of removing all the sources except one and obtaining the voltage across each component, this process will be repeated for each source in the circuit, taking into account the type of source that is been removed, for example, if a voltage source is removed, this will cause a short-circuit, however, if a current source is removed will cause an open circuit, once each circuit with its corresponding source has been analysed, the results will be added to obtain the actual data of the original circuit.

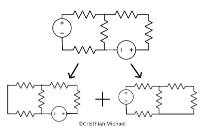

*Figure 4: Superposition example.*

### C. *Transient Response.*

Capacitors are important components in electronic engineering, the function of this component is temporally stored energy obtained from the battery or power supply, to find the transient response of an RC circuit, it is required to obtain the time constant ($\tau$) that is equal to the product of the resistor and the capacitor. (Nahvi, 2002)

$$\tau = RC$$

*Equation 4: Time Constant.*

With the time constant is possible to obtain the graph that represents the charging time of the capacitor until is fully charged.

$$V' = V[1 - e^{\frac{-t}{RC}}]$$

*Equation 5: Capacitor charging.*

It takes around five times constant to reach the maximum voltage of the capacitor, and this voltage will not exceed, the voltage provided by the power supply.

$$Fully\ charge = 5 * \tau$$

*Equation 6: Capacitor fully charge.*

If the battery is removed or the switch of the circuit is removed, the capacitor will be discharged, to obtain the graph of the discharge time the following formula can be used.

$$V' = V[e^{\frac{-t}{RC}}]$$

*Equation 7: Capacitor discharge.*

# III. RESULT

## A. Laboratory 1

Using Thevenin's Theorem, it is possible to simplify complex circuits and to prove that the following circuit has been proposed, this laboratory intends to prove that Thevenin's can be used to analyse circuits, and how its result can be verified in real life by recreating the circuit, obtaining similar values than the ones obtained using the theorem.

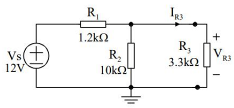


*Figure 5: Thevenin's Equivalent circuit.*

### 1) Calculations

The first step to finding Thevenin's Equivalent is to remove the resistor load, in this case, R3 and then obtain the equivalent voltage through the R2 resistor, which will be the same that the voltage in the open circuit, using the potential divider, the voltage obtained is 10.7V

$$\frac{R_2 * V_{in}}{R_2 + R_1} = V_{OC}\ ;\ \frac{10K * 12}{10K + 1.2K} = 10.7V$$

*Equation 8: Voltage Thevenin's.*

The next step is to obtain the resistor equivalent, to find it, it is required to remove the voltage source and make a short circuit, and then obtain the equivalent of the resistor R1 and R2, obtaining a value of 1071Ω.

$$\frac{R_1 * R_2}{R_1 * R_2} = R_{TH}\ ;\ \frac{1.2K * 10K}{1.2K + 10K} = 1071\Omega$$

*Equation 9: Thevenin's resistance.*

The last step is represented the Equivalent circuit with the resistor load and obtains the voltage and current through it, obtaining a current of *2.45 mA* and a voltage of *8.08V.*

$$\frac{10.71V}{3.3K\,\Omega + 1071\Omega} = 2.45mA$$

*Equation 10: Current.*

$$2.45m * 3.3K = 8.08V$$

*Equation 11: Voltage.*

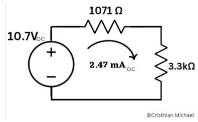


*Figure 6: Thevenin's Equivalent circuit.*

### 2) Proteus

Using a software simulator such as Proteus it's possible to recreate the original circuit and compare the result with Thevenin's equivalent circuit.

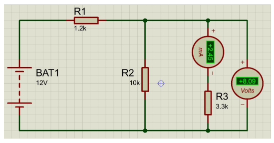


*Figure 7: Original circuit.*

It's possible to observe that the results obtained using Thevenin's Equivalent circuit are the same that the ones obtained in the original circuit.

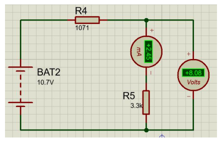


*Figure 8: Thevenin's Equivalent circuit.*

### 3) Breadboard

During the first lab was possible to recreate the circuit and apply Thevenin's using different components.

1. **The equipment and components**
   - Regulated DC supply
   - Mutimeter
   - Breadboard
   - Resistors (*100Ω(×2), 1kΩ(×7), 1.5kΩ, 2.2kΩ(×3), 3.3kΩ, 4.7kΩ and 10kΩ)*

*Figure 9: Components*

The challenge to recreating circuits in real life is that there is a limit of values for resistors, this means that to obtain a specific resistor, different resistors have to be combined to form a close value of the desired resistor for example to obtain the value of $R_1$ equals to 1.2KΩ, three resistors were connected in series to form the 1.2K Ω resistor another challenge it's that real life wires have a small resistance that will affect the final results.

Using the same principles applied before, a voltage of 10.7V was applied to the circuit, however, because there is no resistor with all the values, the Thevenin's Resistor obtained, had to be changed for a close value, in this case, the value of the resistor was 1.1KΩ, this new value will cause new values, however, these values will be close enough.

To obtain the voltage across the load resistor, the voltmeter was connected in parallel obtaining a value of 7.97V and to obtain the current the multimeter is connected in series with the resistor, obtaining 2.4mA, differing with a small difference from the ones obtained in the original circuit, 8.1V and 2.47mA, these values are very close to the ones obtained using proteus and hand calculations.

## B. Laboratory 2

Using nodal analysis and superposition, it is possible to obtain unknown data in circuits with more than one source, this laboratory intends to prove that nodal analysis and superposition are reliable theorems.

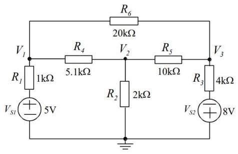


*Figure 10: Original Circuit.*

### 1) Calculations

The first step to solving the circuit using nodal analysis is to find the nodes, once the nodes have been found, it's required to assign an equation to each node depending on if the current is entering or leaving the node.

$$Node\ 1: I_1 - I_4 - I_6 = 0$$
$$Node\ 2: I_4 - I_2 - I_5 = 0$$
$$Node\ 3: I_6 + I_5 - I_3 = 0$$

*Equation 12: Nodes equations.*

Using the equations, the next step is to apply ohm's law to that equation to obtain the voltage of each node.

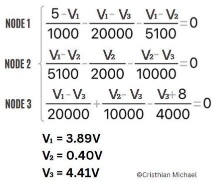


*Figure 11: Nodal calculations.*

Once the equations have been solved, the voltage of each node has been found.

Superposition can be used to find the voltage on each node by removing the voltage sources, studying each voltage source independently, and then adding the results obtained for each voltage source.

For example, removing the voltage source of 8V, and replacing it with a short circuit, the voltage in each node will be different from the ones obtained using nodal analysis, however, to obtain the actual voltage on each node, the same study must be carried out with the voltage source of 5V.

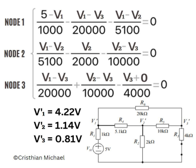


*Figure 12: Superposition and nodal.*

Replacing the voltage source of 5V with a short circuit, the new voltages in each node will be obtained, and with these values, the actual voltage in each node can be obtained.

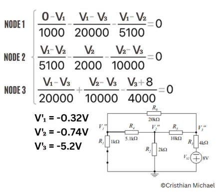


*Figure 13: Superposition and nodal.*

Adding the values obtained in each analysis, it is possible to obtain the real values for each node.

$$V'_1 + V''_1 = V_1\ ;\ 4.22V - 0.32V = 3.9V$$
$$V'_2 + V''_2 = V_2\ ;\ 1.14V - 0.74V = 0.4V$$
$$V'_3 + V''_3 = V_3\ ; 0.81V - 5.2V = 4.39V$$

*Equation 13: Voltage calculations.*

### 2) Proteus

Using Proteus to recreate the circuit, it is possible to identify that the values obtained using nodal analysis are the same as those obtained by using Proteus.

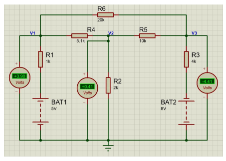

*Figure 14: Original circuit in Proteus.*

If the voltage source of 8V is removed, the voltage in each node is the same that the ones obtained by using the superposition theorem

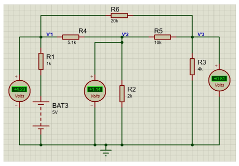

*Figure 15: Superposition with the battery of 5V*

Removing the voltage source of 5V the same results as the ones obtained using the superposition theorem and its calculations.

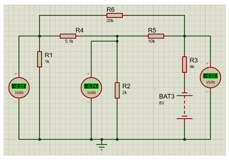

*Figure 16: : Superposition with the battery of 8V.*

### 3) Breadboard

- Regulated DC supply
- Mutimeter
- Breadboard
- Resistors (*1kΩ*(× 3), *1.8kΩ, 2.2kΩ, 5.1kΩ, 10kΩ*(× 3))

*Figure 17: Components.*

Using the components available to recreate this circuit, a few combinations of resistances have been carried out, connecting resistances in series to achieve the desired value, using a regulated DC voltage was possible to provide the correct amount of voltage for the different tests, using the multimeter was possible to obtain similar values than the ones obtained from Proteus or the hand calculations, for example in the first method used the voltage of each node are a bit different between the obtained from the calculations, this is because the wires used have an internal resistance, but this resistance was ignored in the hand calculations.

TABLE I

| Nodal analysis | **V1** | **V2** | **V3** |
|---|---|---|---|
| **Pre Lab** | 3.89 | 0.406 | -4.41 |
| **Lab measures** | 3.85 | 0.4 | -4.39 |

## C. Laboratory 3

### 1) Calculations

To obtain the transfer function of the RC circuit, it's required to calculate the product of the resistor and the capacitor, obtaining a time constant of 1 millisecond.

$$1mS = 100K\Omega * 10nF$$

*Equation 14: Time constant.*

### 2) *Breadboard*

During this lab, different RC waveforms were obtained, depending on the configuration of the circuit or the frequency applied.

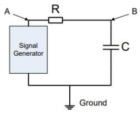


*Figure 18:RC circuit.*

Using a frequency of 300Hz, and a $2V_{p-p}$ amplitude, the time taken for the capacitor to charge a maximum voltage of 1.16V was 1.64 milliseconds, using this data, the time was divided by 12 to represent the graph, it is possible to observe that the time constant for this configuration was, around 600µS this time constant was different from the one obtained before.

TABLE II

| Time | Voltage |
|---|---|
| 0 S | 0V |
| 140 µS | 340 mV |
| 280 µS | 400 mV |
| 400 µS | 520 mV |
| 540 µS | 640 mV |
| 680 µS | 800 mV |
| 820 µS | 880 mV |
| 960 µS | 960 mV |
| 1.1 mS | 1.08V |
| 1.2 mS | 1.12V |
| 1.36 mS | 1.14V |
| 1.5 mS | 1.16V |

Observing the graph obtained it is possible to understand why the time constant was different from the one obtained before, and this is because the capacitor did not reach its maximum value (2V).

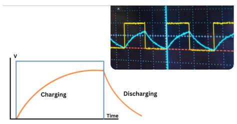


*Figure 19: RC circuit transient response.*

Using the same configuration but changing the frequency to 50Hz, in this case, the time that takes to charge the capacitor is 10 milliseconds, with a maximum voltage of 1.68V, following the same process used before a new table is produced, the time constant has changed as well, and it is new value is almost 0.8 milliseconds.

TABLE III

| Time | Voltage |
|---|---|
| 0 S | 0V |
| 0.80 mS | 1.12V |
| 1.70 mS | 1.48V |
| 2.60 mS | 1.56V |
| 3.40 mS | 1.64V |
| 4.20 mS | 1.64V |
| 5 mS | 1.64V |
| 5.80 mS | 1.65V |
| 6.70 mS | 1.65V |
| 7.60 mS | 1.66V |
| 8.40 mS | 1.68V |
| 9.20 mS | 1.68V |

The new frequency has caused the new transient response to reach a higher voltage, the reason for this is that when the frequency decrease, the capacitance reactance increase, and the current decrease, this effects cause the voltage across the capacitor reaches a higher value, on the other hand, the higher frequency used in the previous test has caused that the capacitance reactance decrease and the current increase, this effect produced a lower voltage in the capacitor.

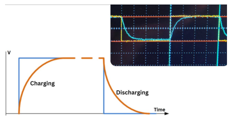

*Figure 20: RC circuit transient response.*

Changing the position of the capacitor with the resistor, and setting a frequency of 200Hz a new transient response is obtained, where the capacitor is charged

TABLE IV

| Time | Voltage |
|---|---|
| 0 | 0 |
| 400 | 1.12V |
| 840 | 720V |
| 1.24 | 480V |
| 1.60 | 400V |
| 2.1 | 240V |
| 2.52 | 200V |
| 2.92 | 720V |
| 3.28 | 1V |
| 3.68 | 1.24V |
| 4.12 | 1.48V |
| 4.52 | 1.56V |

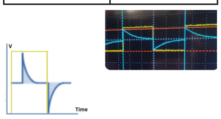

*Figure 21: RC circuit transient response.*

## IV. CONCLUSION

In conclusion, this essay has discussed some of the most common theorems and principles in circuit analysis, through the study of its equations and formulas or with the simulation of these circuits in software applications or with real components, it is possible to observe how these theorems are reliable. Some errors can appear, and the reason for this could be a human error while measuring the data, materials used are not ideal materials for example resistors have a margin of error, and wires in real life cause some resistance, those examples are usually ignored in software applications or hand calculations, even though these are small errors, these theorems are reliable to analyse circuits.

## V. REFERENCES


Hambley, A. R., 2011. *Electrical Engineering, Principles and Applications.* New Jersey: Pearson Education.

Hayt, W. H., 1993. *Engineering Circuit Analysis.* Singapore: McGraw-Hill.

Nahvi, M., 2002. *Electric Circuits.* Fourth ed. New York: McGRAW-HILL.