University of Greenwich

School of Engineering

Analogue Electronics Assignment

ELEE-0052

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Table of Contents

1 TYPE OF FILTERS

1.1 Butterworth.

The Butterworth filters have been designed to have a frequency response as flat as possible in the pass band, obtaining an approximation to an ideal filter [1], to achieve a close approximation to the ideal pass filter an infinite number of reactive components is required, which means that a higher-order filter is needed to obtain a similar frequency response than the expected from an ideal filter. Still, in practice, it is not possible to achieve an ideal frequency response by using a Butterworth, this is because this configuration will produce an excessive increment of ripples in the passband.

This configuration has a roll-off of -20dB/decade in the first order, however, this value changes depending on the order of the filter, for example, a third order would have a -60Db/decade roll-off.

Its characteristics make this filter a good choice for audio-processing applications, for example, they can be used in the design of an audio noise reduction system, for example, microphones work by taking the mechanical energy of a sound wave and turning it into an electrical signal, however, the environment that surrounding the person that is using the environments produce a background noise that depending on the frequency range this noise cannot be heard for the human ears if it is not inside the audible sound frequency range 20Hz to 15KHz [2] however, this noise can affect the audio that will be transmitted by the speakers, in order to reduce this noise, a Butterworth filter can be implemented, by using different reactive components such as capacitors, the transfer function can be adjusted to cancel low or high frequencies by adjusting the component values, and the Sallen-Key configuration used, the most common are the high pass filter (*Fig 1*), and the low pass filter (*Fig 2*) configuration, once the filter has been applied, the noise can be cancel or reduce from the sound wave.

Some of the disadvantages of the Butterworth filters are that they have a slow roll-off compared with other filter designs, which can cause attenuation of frequencies in the stopband and require a large number of components that will increase their value.

1.1.1 High Pass Filter.

$$|H(W)| = \frac{A_0}{\sqrt{1 + (\frac{W_0}{W})^{2n}}}$$

Equation 1: Butterworth high pass filter.

Where:

A_0: Maximum gain in the pass band.

W_0: Lower cut-off frequency.

W: Angular frequency of the input signal.

n: Order of the filter

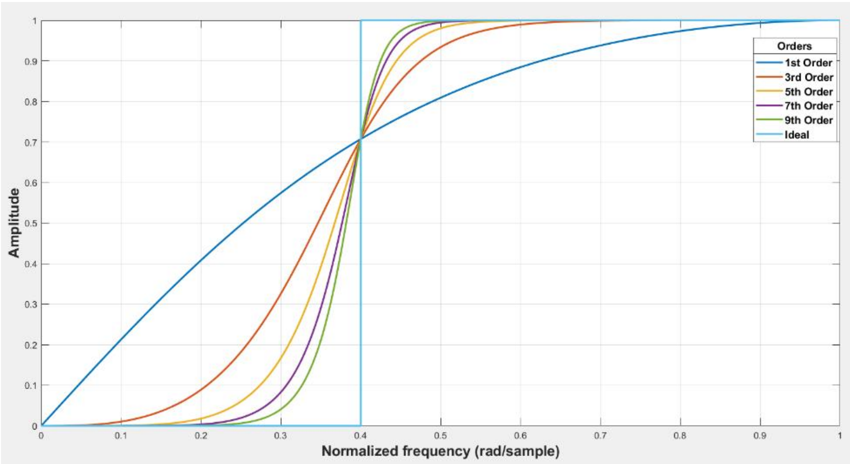


Figure 1: Butterworth High Pass Filter

1.1.2 Low Pass Filter.

$$|H(W)| = \frac{A_0}{\sqrt{1 + (\frac{W}{W_0})^{2n}}}$$

Equation 2: Butterworth low pass filter

Where:

A_0: Maximum gain in the pass band.

W_0: Upper cut-off frequency.

W: Angular frequency of the input signal.

n: Order of the filter

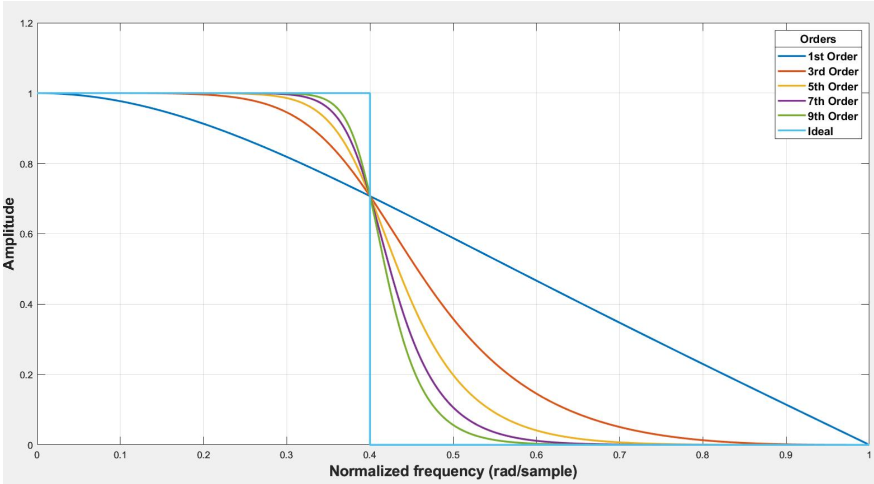


Figure 2: Butterworth Low Pass Filter

1.2 Bessel.

A type of electronic filter called a Bessel filter is used to get a frequency response that is smooth and linear [3] This characteristic can be observed in the following graph (*Fig 3*), some of its applications can be in audio processing or crossover applications, where it can be used to reduce distortion and noise.

A nearly constant group delay across the frequency spectrum is the goal of Bessel filters. This indicates that the filter's phase response is linear because all frequencies in the passband experience the same amount of delay. In audio applications, the linear phase response is important because it helps keep the waveform's shape, which can be important for keeping the signal's quality.

In terms of its frequency response, the Bessel filter has a relatively slow roll-off rate, this means that Bessel filters are less steep than Butterworth filters.

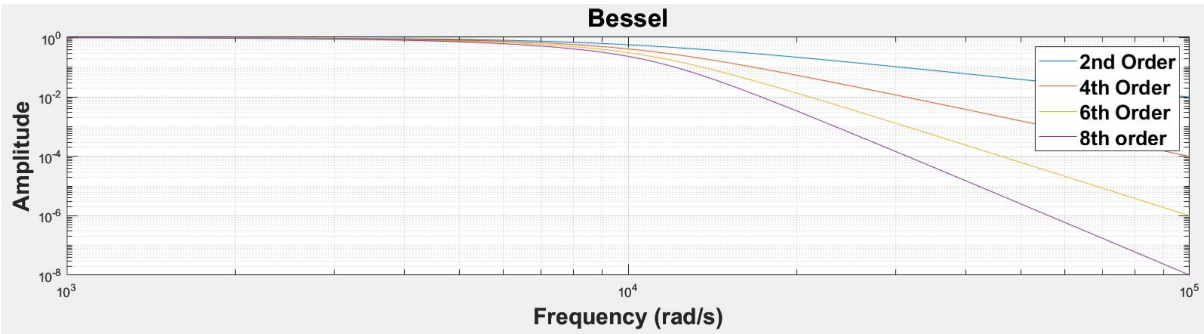


Figure 3:Bessel transfer response.

1.3 Chebyshev.

Chebyshev Type 1 has a steep roll-off rate that makes it possible for them to attenuate unwanted frequencies more quickly than other filters by using fewer components than Butterworth filters. However, the consequence of this steepness is the creation of ripples in the passband (*Fig 4*), indicating that the response of the filter might not be completely flat, the number of ripples and the distances between its peaks change depending on the order of the filter.

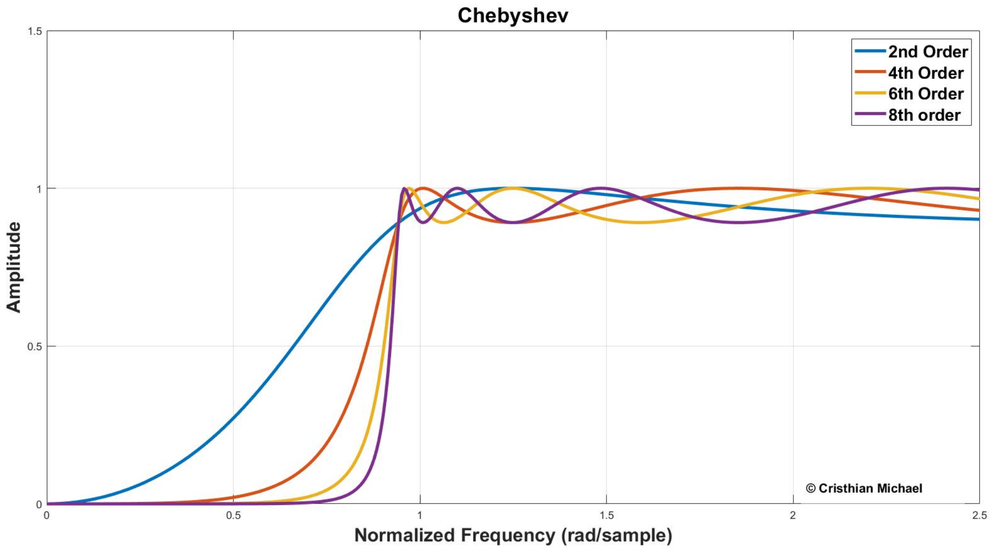


Figure 4: Chebyshev type 1

Similar to Chebyshev Type 1 filters, Chebyshev Type 2 filters have a high roll-off rate. However, the most important difference between these two filters is their frequency response. Chebyshev Type 2, in contrast to Chebyshev Type 1, have a ripple in the stopband (*Fig 5*), allowing certain frequencies in the stopband to pass through, it is frequently utilised in audio and communication applications that require high performance.

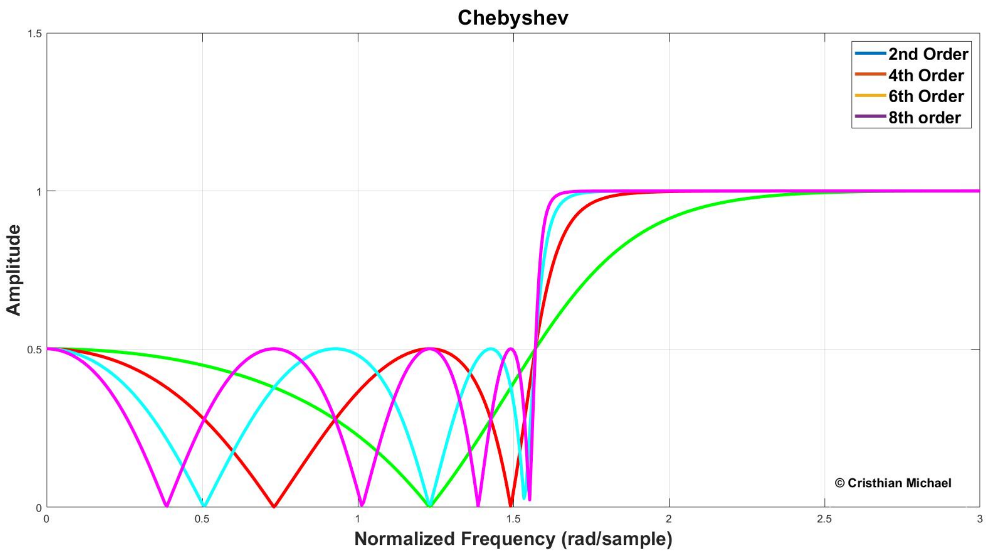


Figure 5: Chebyshev type 2

1.4 Comparison.

The following table shows some of the advantages and disadvantages of these filters [3], however, depending on the application, one filter will be better than the other one.

Butterworth	Advantages	Flat magnitude response
		Pulse response is better than Chebyshev
		attenuation rate better than Bessel
	Disadvantages	Some overshoot and ringing in the step response
Chebyshev	Advantages	Better attenuation beyond the passband than Butterworth
	Disadvantages	Ripple in passband
		ringing in the step response
Bessel	Advantages	smooth step response
		small overshot or ringing
	Disadvantages	Slow roll-off rate

Table 1: Comparison table

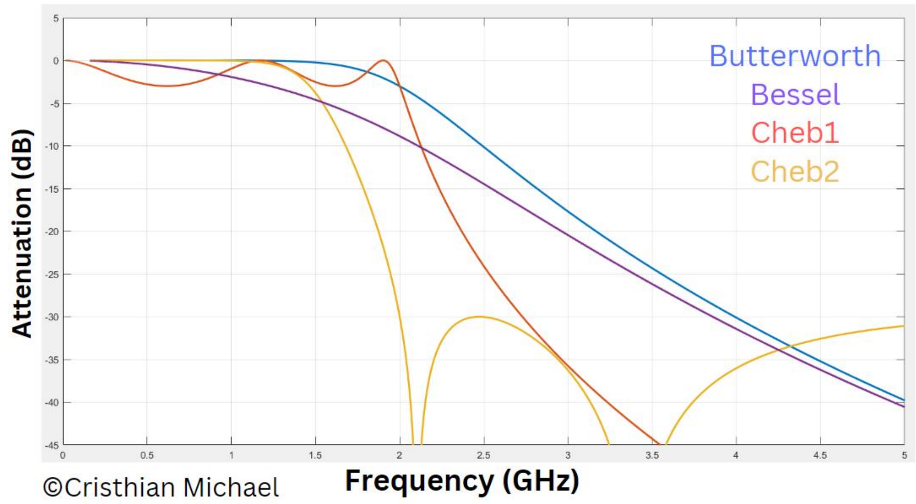


Figure 6: Comparison of the response.

2 SALLEN-KEY SECOND ORDER FILTER.

2.1 Low Pass Filter

The purpose of the Low Pass Filter (*Fig 7*) is to remove or attenuate the high-frequencies and let the low frequencies pass from 0Hz to its cut-off frequency.

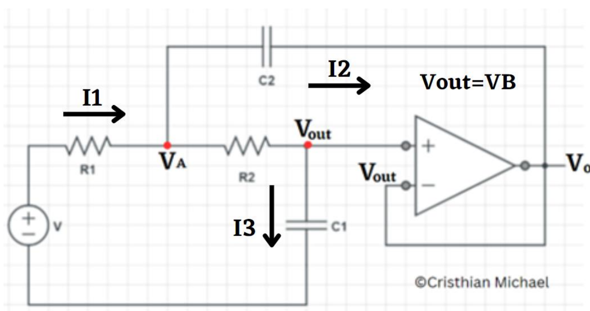


Figure 7: Low pass filter Sallen-Key

The configuration used is a voltage follower, which means that there is no gain in this configuration, the first step is to apply Kirchhoff's current law using the golden rules, it was assumed that no current would flow through the amplifier.

$$I_2 = I_3$$

$$\frac{V_A - V_{out}}{R_2} = \frac{V_{out} - 0}{Z_{C1}}$$

$$\frac{V_A - V_{out}}{R_2} = \frac{V_{out}}{\frac{1}{SC_1}}$$

$$(V_A - V_{out}) = SC_1 * R_2 * V_{out}$$

$$V_A = SC_1 * R_2 * V_{out} + V_{out}$$

$$V_A = V_{out}(SC_1 * R_2 + 1)$$

Equation 3:Node analysis and equation for V_A

$$I_1 = I_2 + I_4$$

$$\frac{V_{in} - V_A}{R_1} = \frac{V_A - V_{out}}{R_2} + \frac{V_A - V_{out}}{Z_{C2}}$$

$$\frac{V_{in}}{R_1} - \frac{V_A}{R_1} = \frac{V_A}{R_2} - \frac{V_{out}}{R_2} + \frac{V_A}{Z_{C1}} - \frac{V_{out}}{Z_{C1}}$$

$$\frac{V_{in}}{R_1} = \frac{V_A}{R_1} + \frac{V_A}{R_2} - \frac{V_{out}}{R_2} + \frac{V_A}{Z_{C1}} - \frac{V_{out}}{Z_{C1}}$$

$$\frac{V_{in}}{R_1} = \frac{V_A}{R_1} + \frac{V_A}{R_2} + \frac{V_A}{Z_{C1}} - \frac{V_{out}}{R_2} - \frac{V_{out}}{Z_{C1}}$$

$$\frac{V_{in}}{R_1} = V_A * \left(\frac{1}{R_1} + \frac{1}{R_2} + SC_1R_1\right) - V_{out} * (\frac{1}{R_2} + SC_1R_1)$$

$$\frac{V_{in}}{R_1} = V_{out}(SC_1 * R_2 + 1) * \left(\frac{1}{R_1} + \frac{1}{R_2} + SC_1R_1\right) - V_{out} * (\frac{1}{R_2} + SC_1R_1)$$

$$\frac{V_{in}}{R_1} = V_{out} * [(SC_1 * R_2 + 1) * \left(\frac{1}{R_1} + \frac{1}{R_2} + SC_1R_1\right) - \left(\frac{1}{R_2} + SC_1R_1\right)]$$

$$\frac{V_{in}}{V_{out}} = R_1 * [(SC_1 * R_2 + 1) * \left(\frac{1}{R_1} + \frac{1}{R_2} + SC_1R_1\right) - \left(\frac{1}{R_2} + SC_1R_1\right)]$$

$$\frac{V_{in}}{V_{out}} = SC_2 * R_2 + 1 + SC_2 * R_1 + S^2 * C_2 * C_1 * R_1 * R_2$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{S^2 * (C_2 * C_1 * R_1 * R_2) + S(C_2 * R_2 + SC_2 * R_1) + 1}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{S^2 * (C_2 * C_1 * R_1 * R_2) + S(C_2 * R_2 + SC_2 * R_1) + 1} * \frac{\frac{1}{C_2 * C_1 * R_1 * R_2}}{\frac{1}{C_2 * C_1 * R_1 * R_2}}$$

$$H(S) = \frac{V_{in}}{V_{out}} = \frac{\frac{1}{R_1 * R_2 * C_1 * C_2}}{S^2 + S\left(\frac{R_1 + R_2}{R_1 * R_2 * C_2}\right) + \frac{1}{R_1 * R_2 * C_1 * C_2}}$$

Equation 4: Low pass filter transfer function

$$H(S) = \frac{V_{in}}{V_{out}} = \frac{\frac{1}{R_1 * R_2 * C_1 * C_2}}{S^2 + S\left(\frac{R_1 + R_2}{R_1 * R_2 * C_2}\right) + \frac{1}{R_1 * R_2 * C_1 * C_2}}$$

Equation 5: Transfer function obtained.

$$H(S) = \frac{a_0}{S^2 + S * a_1 + a_0}$$

Equation 6: Standard transfer function.

2.2 High Pass Filter

The purpose of the High Pass Filter is removed or attenuate the low-frequencies, and letting the high frequencies to pass, it can be used in audio signal processing to remove the low sounds frequencies.

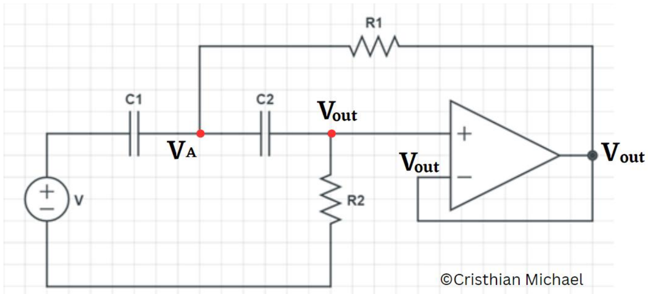


Figure 8: Sallen-Key second order High pass filter configuration

The same operational amplifier was used for the high pass filter, Kirchhoff current law was used to find the equations in the nodes.

$$I_2 = I_3$$

$$\frac{V_A - V_{out}}{Z_{C2}} = \frac{V_{out} - 0}{Z_{R2}}$$

$$\frac{V_A - V_{out}}{\frac{1}{SC_2}} = \frac{V_{out}}{R_2}$$

$$SC_2(V_A - V_{out}) = \frac{V_{out}}{R_2}$$

$$V_A = \frac{V_{out}}{R_2 * SC_2} + V_{out}$$

$$V_A = V_{out}(\frac{1}{R_2 * SC_2} + 1)$$

Equation 7: Node analysis and equation for V_A.

$$I_1 = I_2 + I_4$$

$$\frac{V_{in} - V_A}{\frac{1}{SC_1}} = \frac{V_A - V_{out}}{\frac{1}{SC_2}} + \frac{V_A - V_{out}}{R_1}$$

$$SC_1(V_{in} - V_A) = SC_2(V_A - V_{out}) + \frac{V_{A-}V_{out}}{R_1}$$

$$V_{in} - V_A = \frac{SC_2 * V_A + \frac{V_A}{R_1} - SC_2 * V_{out} - \frac{V_{out}}{R_1}}{SC_1}$$

$$V_{in} - V_A = \frac{V_A * (SC_2 + \frac{1}{R_1}) - V_{out}(SC_2 + \frac{1}{R_1})}{SC_1}$$

$$V_{in} = \frac{V_A * (SC_2 + \frac{1}{R_1}) - V_{out}(SC_2 + \frac{1}{R_1})}{SC_1} + V_A$$

$$V_{in} = \frac{V_{out} * (\frac{1}{R_2 * SC_2} + 1) * (SC_2 + \frac{1}{R_1}) - V_{out}(SC_2 + \frac{1}{R_1})}{SC_1} + V_{out}\left(\frac{1}{R_2 * SC_2} + 1\right)$$

$$V_{in} = V_{out}\left(\frac{SC_2 * R_2 + 1}{S^2 C_1 * C_2 * R_1 * R_2} + \frac{1}{S^2 C_2 * R_2} + 1\right)$$

$$\frac{V_{0ut}}{V_{in}} = \frac{S^2}{S^2 + \frac{S}{C_2 * R_2} + \frac{SC_2 * R_2}{C_1 * C_2 * R_1 * R_2}) + \frac{1}{C_1 * C_2 * R_1 * R_2}}$$

$$\frac{V_{0ut}}{V_{in}} = \frac{S^2}{S^2 + S(\frac{1}{C_2 * R_2} + \frac{1}{C_1 * R_2}) + \frac{1}{C_1 * C_2 * R_1 * R_2}}$$

Equation 8: High pass filter derivation

$$H(S) = \frac{V_{0ut}}{V_{in}} = \frac{S^2}{S^2 + S(\frac{1}{C_2 * R_2} + \frac{1}{C_1 * R_2}) + \frac{1}{C_1 * C_2 * R^2}}$$

Equation 9: Transfer function obtained.

$$H(S) = \frac{S^2}{S^2 + S * a_1 + a_0}$$

Equation 10: Standard transfer function.

2.3 Second Order Filter Design

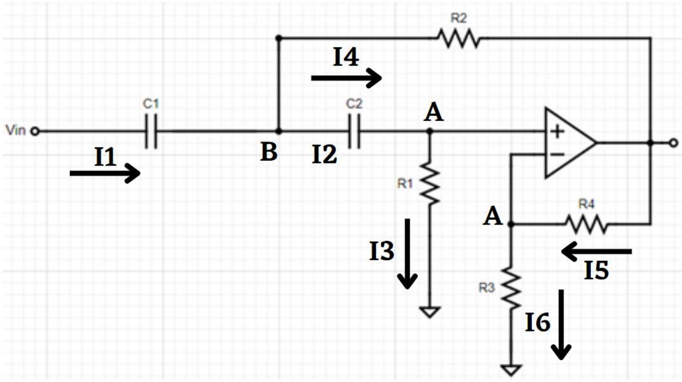


Figure 9: Second order high pass filter with gain

For this exercise a non-inverted configuration was used for the operational amplifier, with this configuration it was possible to obtain a gain, in order to obtain the equation for the gain, nodal analysis and the golden rules for an ideal amplifier were used, formulating the equation for that node where R_3 and R_4 were implemented, it was possible to obtain the gain equation.

$$I_5 = I_6$$

$$\frac{V_{out} - V_A}{R_4} = \frac{V_A - 0}{R_3}$$

$$\frac{V_{out} - V_A}{R_4} = \frac{V_A}{R_3}$$

$$V_{out} - V_A = \frac{V_A * R_4}{R_3}$$

$$V_{out} = \frac{V_A * R_4}{R_3} + V_A$$

$$V_{out} = V_A * (\frac{R_4}{R_3} + 1)$$

$$\frac{1}{V_A} = \frac{(\frac{R_4}{R_3} + 1)}{V_{out}}$$

$$V_A = \frac{R_3 * V_{out}}{R_4 + R_3}$$

$$V_A = V_{out}(\frac{R_3}{R_4 + R_3})$$

Equation 11: Node analysis and equation for V_A .

With the gain equation obtained, it was required to apply nodal analysis in the node A and node B to obtain the transfer function.

$$I_1 = I_2 + I_4$$
$$I_2 = I_3$$

Equation 12: KCL equations.

$$\frac{V_{in} - V_B}{Z_{C1}} = \frac{V_B - V_A}{Z_{C2}} + \frac{V_B - V_{out}}{Z_{R2}}$$
$$\frac{V_B - V_A}{Z_{C2}} = \frac{V_A - 0}{Z_{R1}}$$

Equation 13: KCL equations to obtain the transfer function.

$$H(S) = \frac{V_{0ut}}{V_{in}} = \frac{A * S^2}{S^2 + S(\frac{1}{C_2 * R_1} + \frac{1}{C_1 * R_1} - \frac{R_4}{R_2 * R_3 * C_1}) + \frac{1}{C_1 * C_2 * R_1 * R_2}}$$

Equation 14: Transfer function obtained.

The gain in amplifiers represents how much the input signal has been amplify, for this task, a gain of 27dB is required, to convert from decibel to linear gain the following equation was used.

$$Gain = 20logA$$
$$27dB = 20logA$$
$$\frac{27}{20} = logA$$
$$10^{\frac{27}{20}} = A$$
$$A = 22.40$$

Equation 15: dB Gain to linear Gain workout

The linear gain equivalent to 27dB is equal to 22.40, using this gain, it is possible to find the resistors values by using the gain equation obtained for the second order high pass filter.

$$Gain = \frac{R_4}{R_3} + 1$$

Equation 16: Gain equation

Substituting the linear gain into the equation, it is possible to observe that the gain depends on the values of the resistors (*R3 and R4*), to simplify the equation, the value of one of the resistors is assumed, for this exercise, the value given to R_4 is 22KΩ, this left only one variable in the equation, obtaining a value of 1028Ω for the R_3, however to make the circuit more realistic, values from the E24 list are used for all the resistor of this exercise, this means that the new value for R_3 is 1kΩ.

$$22.40 - 1 = \frac{R_4}{R_3} \ ; \ 21.40 = \frac{R_4}{R_3}$$

$$R_4 = 22K\Omega$$

$$R_3 = \frac{22K}{21.40}$$

$$R_3 = 1028\Omega \ ; \ R_3 \approx 1K\Omega$$

Equation 17: R3 and R4 values workout

With the resistors obtained, a linear gain of 23 was found, this value is a bit higher than the ideal gain that the amplifier should produce which is 22.40, however this value can be accepted depending on the application or accuracy required, if a high accuracy is required, two resistors can be placed in series to obtain and close resistor value for R_3, for example using two resistors in series with the values of 560Ω and 470Ω obtaining 1030Ω which is quite similar to the actual value desired for R_3 obtaining a gain of 22.60 which is closer to the gain that is expected.

$$Gain \approx \frac{22K\Omega}{1K\Omega} + 1$$

$$Gain = K \approx 23$$

Equation 18: Gain obtained.

Once the values of R_4 and R_3 have been found, it is possible to start working on the cut-off frequency of the systems, however, one of the characteristics of a high pass filter, it is that the quality factor must be positive, this characteristic is quite important at the moments of choosing the values for the resistor and capacitors, to simplify the calculations, the values for C_1 and C_2 have been set up to be equals, and the resistors R_1 and R_2, using the equation obtained for the quality factor [4], another assumption has been done, selecting the same quality factor used for a Butterworth, was possible to find the x value that will determinate the value of R_1.

$$C_1 = C_2 = C\,;\ R_1 = xR\,;\ R_2 = R$$

Equation 19: Values assumption

$$Q = \frac{\sqrt{C_1 * C_2 * R_1 * R_2}}{R_2 * C_2 + R_2 * C_1 + R_1 * C_2\,(1 - \mathrm{K})}$$

$$Q = \frac{CR\sqrt{x}}{R * C + R * C + xR * C\,(1 - \mathrm{K})}$$

$$Q = \frac{C * R\sqrt{x}}{R * C(2 + x\,(1 - \mathrm{K}))}\ ;\ Q = \frac{\sqrt{x}}{2 + x(1 - \mathrm{K})}$$

$$Q = \frac{\sqrt{x}}{2 + x(1 - 23)}\ ;\ 0{,}707 = \frac{\sqrt{x}}{2 + x(-22)}$$

$$x = 0.073$$

Equation 20: Factor x for a 0.707 quality factor

Once the x-factor that will determine the value for the resistor is found, it's possible to start working on the cut-off frequency, for this exercise, the cut-off frequency required is 15KHz, using the equation obtained, it is possible to work out in value of the components that will be used to achieve the cut-off frequency.

$$f_c = \frac{1}{2\pi\sqrt{C_1 C_2 R_1 R_2}}$$

Equation 21: Cut-off frequency equation

The relation stablish in equation for the capacitors will be used to determine the value of the resistors, giving a value of 1.5nF to the capacitors, it's possible to obtain the value resistor.

$$C_1 = C_2 = 1.5nF$$

$$R_1 = xR$$

$$R_2 = R$$

Equation 22: Component values.

$$f_c = \frac{1}{2\pi\sqrt{C * C * R * xR}}$$

$$f_c = 15kHz$$

$$15KHz = \frac{1}{2 * \pi * 1.5n * R * \sqrt{0.073}}$$

$$R = 26180\Omega$$

$$R_2 = R = 18000 + 8200$$

$$R_2 \approx 26200$$

$$R_1 = xR \;;\quad R_1 = 0.073 * 26180$$

$$R_1 = 1911$$

$$R_1 = 1000\Omega \; + \; 910\Omega$$

$$R_1 \approx 1910\Omega$$

Equation 23: R1 and R2 values workout.

Now that all the components have been calculated, it is possible to place the values in the second-order transfer function previously obtained.

Components	Value
R1	1.91KΩ
R2	26.2KΩ
R3	1KΩ
R4	22KΩ
C1	1.5nF
C2	1.5nF

Table 2: Value of components

$$H(S) = \frac{V_{0ut}}{V_{in}} = \frac{23 * S^2}{S^2 + S(\frac{1}{1.5n * 1.91K} + \frac{1}{1.5n * 1.91K} - \frac{22K}{26.2K * 1K * 1.5n}) + \frac{1}{1.5n * 1.5n * 1.91K * 26.2K}}$$

$$H(S) = \frac{V_{0ut}}{V_{in}} = \frac{23 * S^2}{S^2 + S\left(1.23 * 10^5\right) + 8.23 * 10^9}$$

Equation 24: Transfer Function

Plotting the transfer function in MATLAB with the component's values obtained previously, was possible to display the Bode Plot of the filter, some important characteristics of the filter obtained is that the cut-off frequency in the phase is almost 90 degrees, the small error can be caused for the value of the components used, because some of the value used for the capacitors and resistors are commercial values and not ideal components value, the same problem appears when checking the gain at the cut-off frequency with a value of 23.9Db instead of 24dB.

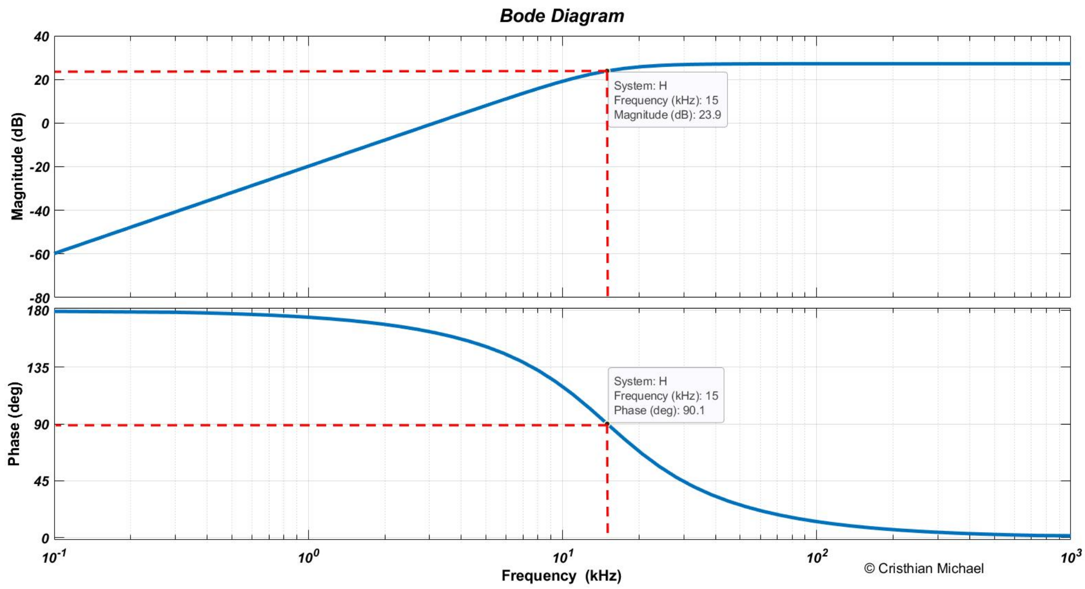


Figure 10: Bode plot.

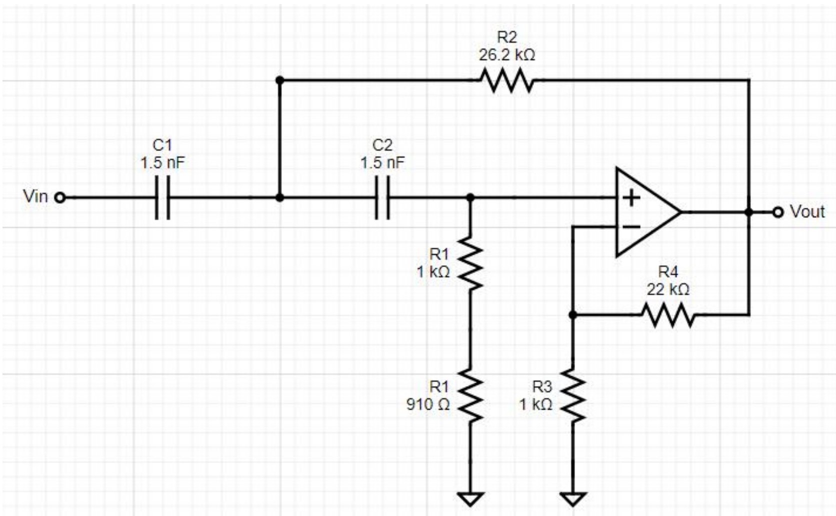


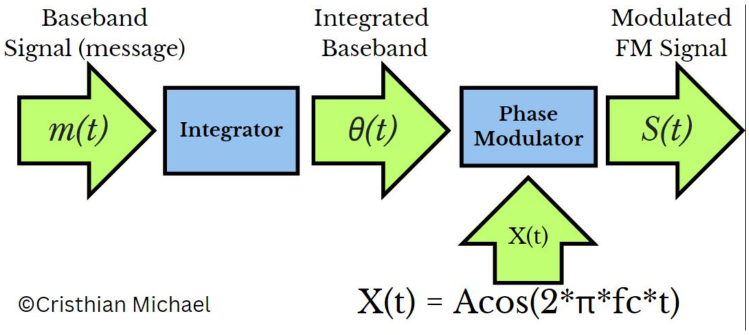
Figure 11: Sallen-Key configuration with the components obtained.

3 FREQUENCY MODULATION AND DEMODULATION.

3.1 Modulation

Modulation is the process by which an unmodulated carrier signal that is varied in accordance with modulating the input signal, the function of the carrier is to vary linearly with the input signal *m(t)* in other words, the carrier signal changes its frequency depending on the amplitude of input signal, without change its frequency.

The first step would be representing the message or baseband *m(t)* that will be transmitted, the integration with respect to time will be calculated for this signal, obtaining an equation for the phase with respect to time, then an unmodulated carrier signal with a constant amplitude and frequency will be add in the process resulting in a frequency modulated signal or FM signal [3].



$$C(t) = A_C * \cos\left(w_C t + \theta\right)$$

Equation 25: Mathematical expression for the carrier.

Where:

A_C: Carrier amplitude.

W_c: Angular frequency of the carrier (radians/sec) = $2\pi f_C$.

f_c: Carrier frequency in Hz.

$$C(t) = A_C * \cos\left(2\pi f_C t\right)$$

$$\frac{d}{dt}\theta = \frac{d}{dt}\left(2\pi f_C t\right)$$

$$\frac{d}{dt}\theta = 2\pi f_C$$

$$\frac{d}{dt}\theta = W_C$$

Equation 26: Angular frequency relation with the phase angle

$$FM(t) = A_C * \cos(\theta)$$

$$\theta_i = 2\pi f_i t$$

$$\frac{d}{dt}\theta_i = \frac{d}{dt}(2\pi f_i t)$$

$$\frac{d}{dt}\theta_i = 2\pi f_i$$

$$\frac{d}{dt}\theta_i = w_i$$

Equation 27: Instantaneous angular frequency relation with the phase angle

$$W_i = W_C + K_f * m(t)$$

Equation 28: Instantaneous angular frequency of the carrier

W_i: Angular frequency of the carrier.
W_c: Angular frequency of the carrier (radians/sec) = $2\pi f_C$.
K_f: Frequency sensitivity constant in Hz/Volt.
m(t): Message or input signal

$$\frac{d}{dt}\theta_i = W_C + K_f * m(t)$$

$$d\theta_i = \left(W_C + K_f * m(t)\right)dt$$

$$\int d\theta_i = \int \left(W_C + K_f * m(t)\right)dt$$

$$\int d\theta_i = \int W_C dt + \int K_f * m(t)dt$$

$$\theta_i = W_C t + K_f \int m(t)dt$$

.

Equation 29: Integration

$$FM(t) = A_C * \cos\left(W_C t + K_f \int m(t)dt\right)$$

$$m(t) = A_m \cos W_m t$$

$$FM(t) = A_C * \cos\left(W_C t + K_f \int A_m \cos W_m t \, dt\right)$$

$$FM(t) = A_C * \cos\left(W_C t + \frac{K_f * A_m}{W_m} \sin W_m t\right)$$

$$K_f * A_m = \Delta_f = Frequency\ deviation$$

$$m_f = \frac{\Delta_f}{f_m} = \frac{\Delta_w}{W_m}$$

$$FM(t) = A_C * \cos\left(W_C t + m_f * \sin W_m t\right)$$

Equation 30: Single-tone FM wave

3.2 Demodulation.

FM demodulation is one of the main processes in the reception of frequency modulation signals, the process consists of receiving and filtering the signal obtained to recover the initial message signal by removing the carrier signal, one of the techniques used is called Phase Locked Loop.

The Phase Locked Loop is used to track the phase and frequency of the carrier signal inside the modulated signal instead of tracking the amplitude, the operation of this method consists of following the input signal using a feedback loop, where the fed-back signal has to be equal to the input signal, otherwise the error signal will change the value for the fed back until they both equal, the error produced is used to adjust the voltage controlled oscillator frequency by approaching the values of the instantaneous phase angle to the angle of the incoming signal, causing the synchronisation of both signals

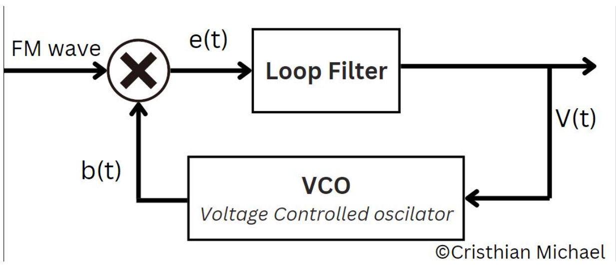


Figure 12: Closed-loop for demodulation

3.3 Applications.

Frequency modulation also known as FM is commonly used for radio and television broadcast, in radio broadcast, the frequency range that operates goes from 88 MHz to 108 M studying the behaviour or use of a FM radio [2], it is possible to explain how FM works in these devices, humans can detect sounds from 20Hz to 20KHz, however, not all the humans can hear those frequencies, FM radio works with maximum frequency modulation of 15KHz [4] , another important parameter is the maximum frequency deviation that FM radio usually uses 75KHz, using these values a deviation ratio of 5 is obtained.

$$m = \frac{\max frequency\ deviation}{\max frequency\ modulation}$$

$$m = \frac{75\ KHz}{15\ KHz} \quad ; \quad m = 5$$

Equation 31: Deviation ratio

Using Carson's rule [5], it's possible to determinate the bandwidth, obtaining a value of 180KHz.

$$Bandwidth = 2 * (\max frequency\ deviation - \max frequency\ modulation)$$

$$B = 2 * (75KHz + 15KHz) \quad ; \quad B = 180KHz$$

Equation 32: Bandwidth calculation.

Once the bandwidth is found, a guard band of 20KHz is added to it, this prevents interferences by separating two frequency range to ensure that both can transmit simultaneously without interferences, this is called channel bandwidth and it has a value of 200KHz.

$$Channel\ Bandwidth = 180KHz + 20KHz \quad ; \quad Channel\ Bandwidth = 200KHz$$

Equation 33: Channel Bandwidth

Using the bandwidth of 200KHz, it is possible to obtain the number of channels that can be used in the FM frequency range, obtaining a maximum of 100 channels.

$$N\ Channels = \frac{20000KHz}{200KHz} \quad ; \quad N\ Channels = 100$$

Equation 34: Number of channels.

Using the limit values for the FM frequency range was possible to calculate the wavelength [6], by dividing the frequency respect to the speed of light, obtaining a minimal value of 2.7m and a maximum of 3.4m.

$$\lambda = \frac{3 * 10^8}{Frequency}$$

$$\lambda = \frac{3 * 10^8}{88 * 10^6} \ ; \ \lambda = 3.4m$$

$$\lambda = \frac{3 * 10^8}{108 * 10^6} \ ; \ \lambda = 2.7m$$

Equation 35: Wavelength.

3.4 Advantages and Disadvantages

The use of frequency modulation is well spread around the world, used in many applications, in the follow table [4], some of its advantages and disadvantages can be observed.

Advantages	Disadvantages
Less interference and noise.	Equipment cost is higher.
Power Consumption is less as compared to AM.	More complicated receiver and transmitter
Adjacent FM channels are separated by guard bands.	The antennas for FM systems should be kept close for better communication.
Improved system fidelity	large bandwidth

Table 3: Advantages and Disadvantages.

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5 Appendix

Workout

$$I_2 = I_3 \to \frac{V_A - V_0}{Z_{C2}} = \frac{V_0 - 0}{Z_{R2}} \to \frac{V_A - V_0}{\frac{1}{sC_2}} = \frac{V_0}{R_2} \to sC_2(V_A - V_0) = \frac{V_0}{R_2} \to V_A - V_0 = \frac{V_0}{sC_2R_2}$$

$$V_A = \frac{V_0}{R_2sC_2} + V_0 \to V_A = V_0\left(\frac{1}{R_2sC_2} + 1\right)$$

$$I_1 = I_2 + I_4$$

$$\frac{V_1 - V_A}{\frac{1}{sC_1}} = \frac{V_A - V_0}{\frac{1}{sC_2}} + \frac{V_A - V_0}{R_1} \to sC_1(V_1 - V_A) = sC_2(V_A - V_0) + \frac{V_A - V_0}{R_1} \to V_1 - V_A = \frac{sC_2(V_A - V_0) + \frac{V_A - V_0}{R_1}}{sC_1}$$

$$V_1 - V_A = \frac{sC_2V_A + \frac{V_A}{R_1} - sC_2V_0 - \frac{V_0}{R_1}}{sC_1} \to V_1 - V_A = \frac{V_A(sC_2 + \frac{1}{R_1}) - V_0(sC_2 + \frac{1}{R_1})}{sC_1}$$

$$V_1 = \frac{V_A(sC_2 + 1/R_1) - V_0(sC_2 - 1/R_1)}{sC_1} + V_A \to V_1 = \frac{\left(\frac{V_0}{R_2sC_2} + V_0\right)(sC_2 + 1/R_1) - V_0(sC_2 + 1/R_1)}{sC_1} + \frac{V_0}{sC_2R_2} + V_0$$

$$V_1 = V_0\left(\frac{\left(\frac{1}{sC_2R_2} + 1\right)(sC_2 + 1/R_1) - (sC_2 + 1/R_1)}{sC_1} + \frac{1}{R_2sC_2} + 1\right)$$

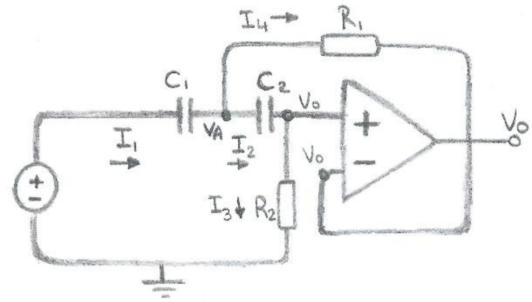
$$V_1 = V_0\left(\frac{\frac{sC_2}{sC_2R_2} + sC_2 + \frac{1}{sC_2R_2R_1} + \frac{1}{R} - sC_2 - 1/R_1}{sC_1} + \frac{1}{sC_2R_2} + 1\right) \to V_1 = V_0\left(\frac{\frac{sC_2}{sC_2R_2} + \frac{1}{sC_2R_2R_1}}{sC_1} + \frac{1}{sC_2R_2} + 1\right)$$

$$V_1 = V_0\left(\frac{\frac{sC_2R_2 + 1}{sC_2R_2R_1}}{sC_1} + \frac{1}{sC_2R_2} + 1\right) \to V_1 = V_0\left(\frac{sC_2R_2 + 1}{s^2C_1C_2R_2R_1} + \frac{1}{sC_2R_2} + 1\right) \to \frac{V_1}{V_0} = \left(\frac{sC_2R_2 + 1}{s^2C_1C_2R_1R_2} + \frac{1}{sC_2R_2} + 1\right)$$

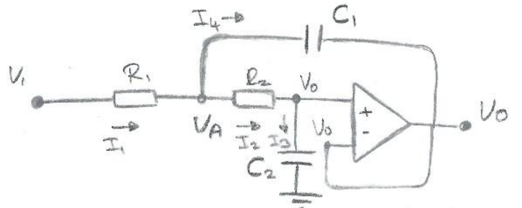
$$\frac{V_0}{V_1} = \frac{1}{\frac{sC_2R_2 + 1}{s^2C_1C_2R_1R_2} + \frac{1}{sC_2R_2} + 1} \to \frac{V_0}{V_1} = \frac{1}{\frac{sC_2R_2 + 1}{s^2C_1C_2R_1R_2} + \frac{1}{sC_2R_2} + 1} \cdot \frac{s^2}{s^2} \to \frac{V_0}{V_1} = \frac{s^2}{\frac{s^3C_2R_2 + s^2}{s^2C_1C_2R_2R_1} + \frac{s^2}{sC_2R_2} + s^2}$$

$$H(s) = \frac{V_0}{V_1} = \frac{s^2}{\frac{sC_2R_2 + 1}{C_1C_2R_2R_1} + \frac{s}{C_2R_2} + s^2} \to H(s) = \frac{s^2}{s^2 + \frac{s}{C_2R_2} + \frac{sC_2R_2}{C_1C_2R_2R_1} + \frac{1}{C_1C_2R_2^2}}$$

$$H(s) = \frac{s^2}{s^2 + s\left(\frac{1}{C_2R_2} + \frac{1}{C_1R_2}\right) + \frac{1}{C_1C_2R_2R_1}}$$



High Pass Filter Derivation Workout



$$I_2 = I_3 \rightarrow \frac{V_A - V_0}{R_2} = \frac{V_0 - 0}{Z_{C2}} \rightarrow \frac{V_A - V_0}{R_2} = \frac{V_0}{Z_{C2}} \rightarrow V_A - V_0 = \frac{V_0 R_2}{Z_{C2}} \rightarrow V_A = \frac{V_0 R_2}{Z_{C2}} + V_0$$

$$V_A = \frac{V_0 R_2}{\frac{1}{sC_2}} + V_0$$

$$V_A = sC_2(V_0 R_2) + V_0$$

$$V_A = V_0(sC_2R_2 + 1)$$

$$I_1 = I_2 + I_4 \rightarrow \frac{V_{in} - V_A}{R_1} = \frac{V_A - V_0}{R_2} + \frac{V_A - V_0}{Z_{C1}}$$

$$\frac{V_1 - V_A}{R_1} = \frac{V_A}{R_2} - \frac{V_0}{R_2} + \frac{V_A}{Z_{C1}} - \frac{V_0}{Z_{C1}}$$

$$\frac{V_1}{R_1} - \frac{V_A}{R_1} = \frac{V_A}{R_2} + \frac{V_A}{Z_{C1}} - \frac{V_0}{R_2} - \frac{V_0}{Z_{C1}}$$

$$\frac{V_1}{R_1} = \frac{V_A}{R_1} + \frac{V_A}{R_2} + \frac{V_A}{Z_{C1}} - \frac{V_0}{R_2} - \frac{V_0}{Z_{C1}} \rightarrow \frac{V_1}{R_1} = V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_1\right) - V_0\left(\frac{1}{R_2} + sC_1\right)$$

$$V_1 = V_A\left(\frac{R_1}{R_1} + \frac{R_1}{R_2} + sC_1R_1\right) - V_0\left(\frac{R_1}{R_2} + sC_1R_1\right)$$

$$V_i = V_0(sC_2R_2 + 1)\left(1 + \frac{R_1}{R_2} + sC_1R_1\right) - V_0\left(\frac{R_1}{R_2} + sC_1R_1\right)$$

$$V_i = V_0\left(sC_2R_2 + 1 + \frac{sC_2R_2R_1}{R_2} + \frac{R_1}{R_2} + s^2C_1C_2R_1R_2 + sC_1R_1\right) - V_0\left(\frac{R_1}{R_2} + sC_1R_1\right)$$

$$\frac{V_1}{V_0} = sC_2R_2 + 1 + sC_2R_1 + \cancel{\frac{R_1}{R_2}} + s^2C_1C_2R_1R_2 + sC_1R_1 - \cancel{\frac{R_1}{R_2}} - sC_1R_1$$

$$\frac{V_1}{V_0} = sC_2R_2 + 1 + sC_2R_1 + s^2C_1C_2R_1R_2 \rightarrow \frac{V_1}{V_0} = s^2(C_1C_2R_1R_2) + s(C_2 + C_2R_2) + 1$$

$$\frac{V_0}{V_1} = \frac{1}{s^2(C_1C_2R_1R_2) + s(C_2R_1 + C_2R_2) + 1} \rightarrow$$

$$H(s) = \frac{1}{s^2(C_1C_2R_1R_2) + s(C_2R_1 + C_2R_2) + 1} \cdot \frac{\frac{1}{C_1C_2R_1R_2}}{\frac{1}{C_1C_2R_1R_2}}$$

$$H(s) = \frac{\frac{1}{C_1C_2R_1R_2}}{s^2 + s\frac{(C_2R_1 + C_2R_2)}{C_1C_2R_1R_2} + \frac{1}{C_1C_2R_1R_2}} \rightarrow H(s) = \frac{\frac{1}{C_1C_2R_1R_2}}{s^2 + s\left(\frac{R_1 + R_2}{R_1R_2C_1}\right) + \frac{1}{R_1R_2C_1C_2}}$$

Low Pass Filter Derivation Workout

$K=\left(\frac{R_4}{R_3}+1\right) \rightarrow V_A=\frac{V_0 R_3}{R_4+R_3}$ or $\frac{V_0}{K}$

$$① \; B \rightarrow I_{C_1}=I_{C_2}+I_{R_2} \rightarrow \frac{V_1-V_B}{Z_{C_1}}=\frac{V_B-V_A}{Z_{C_2}}+\frac{V_B-V_0}{Z_{R_2}} \rightarrow \frac{V_1}{Z_{C_1}}=\frac{V_B}{Z_{C_2}}+\frac{V_B}{Z_{C_1}}+\frac{V_B}{Z_{R_2}}-\frac{V_A}{Z_{C_2}}-\frac{V_0}{Z_{R_2}}$$

$$① \; \frac{V_1}{Z_{C_1}}=V_B\left(\frac{1}{Z_{C_1}}+\frac{1}{Z_{R_2}}+\frac{1}{Z_{C_2}}\right)-V_A\left(\frac{1}{Z_{C_2}}\right)-\frac{V_0}{Z_{R_2}}$$

$$① \; \frac{V_1}{Z_{C_1}}=V_B\left(\frac{Z_{R_2}Z_{C_2}+Z_{C_1}Z_{C_2}+Z_{C_1}Z_{R_2}}{Z_{C_1}Z_{R_2}Z_{C_2}}\right)-\frac{V_0}{K}\left(\frac{1}{Z_{C_2}}\right)-\frac{V_0}{Z_{R_2}}$$

$$① \; \frac{V_1}{Z_{C_1}}=V_B\left(\frac{Z_{R_2}Z_{C_2}+Z_{C_1}Z_{C_2}+Z_{C_1}Z_{R_2}}{Z_{C_1}Z_{C_2}Z_{R_2}}\right)-\frac{V_0}{KZ_{C_2}}-\frac{V_0}{Z_{R_2}}$$

$$② \; I_{C_2}=I_{R_1} \rightarrow \frac{V_B-V_A}{Z_{C_2}}=\frac{V_A-0}{Z_{R_1}} \rightarrow \frac{V_B}{Z_{C_2}}=\frac{V_A}{Z_{R_1}}+\frac{V_A}{Z_{C_2}} \rightarrow V_B=V_A \cdot Z_{C_2}\left(\frac{1}{Z_{R_1}}+\frac{1}{Z_{C_2}}\right)$$

$$V_B=V_A\left(\frac{Z_{C_2}}{Z_{R_1}}+1\right) \rightarrow V_B=\frac{V_0}{K}\left(\frac{Z_{C_2}}{Z_{R_1}}+1\right)$$

$$② \; V_B=\frac{V_0 Z_{C_2}}{KZ_{R_1}}+\frac{V_0}{K}$$

mix → 1 and 2

$$\frac{V_1}{Z_{C_1}}=\left(\frac{V_0 Z_{C_2}}{KZ_{R_1}}+\frac{V_0}{K}\right)\left(\frac{Z_{R_2}Z_{C_2}+Z_{C_1}Z_{C_2}+Z_{C_1}Z_{R_2}}{Z_{C_1}Z_{C_2}Z_{R_2}}\right)-\frac{V_0}{KZ_{C_2}}-\frac{V_0}{Z_{R_2}}$$

$$\frac{V_1}{Z_{C_1}}=V_0\left(\frac{Z_{C_2}}{KZ_{R_1}}+\frac{1}{K}\right)\left(\frac{Z_{R_2}Z_{C_2}+Z_{C_1}Z_{C_2}+Z_{C_1}Z_{R_2}}{Z_{C_1}Z_{C_2}Z_{R_2}}\right)-\frac{1}{KZ_{C_2}}-\frac{1}{Z_{R_2}}$$

$$\frac{V_1}{Z_{C_1}\cdot V_0}=\left(\frac{Z_{R_2}Z^2_{C_2}+Z_{C_1}Z^2_{C_2}+Z_{C_1}Z_{C_2}R_2}{KZ_{R_1}Z_{R_2}Z_{C_1}Z_{C_2}}+\frac{Z_{R_2}Z_{C_2}+Z_{C_1}Z_{C_2}+Z_{C_1}Z_{R_2}}{KZ_{C_1}Z_{C_2}Z_{R_2}}\right)-\left(\frac{Z_{R_2}+KZ_{C_2}}{KZ_{C_2}Z_{R_2}}\right)$$

$$\frac{V_1}{Z_{C_1}\cdot V_0}=\left(\frac{Z_{R_2}Z^2C_2+Z_{C_1}Z^2C_2+Z_{C_1}Z_{C_2}R_2+Z_{R_2}Z_{C_2}Z_{R_1}+Z_{C_1}Z_{R_2}Z_{R_1}+Z_{C_1}Z_{R_2}Z_{R_1}}{KZ_{C_1}Z_{C_2}Z_{R_2}Z_{R_1}}\right)-\left(\frac{Z_{R_2}+KZ_{C_2}}{KZ_{C_2}Z_{R_2}}\right)$$

$$\frac{V_1}{Z_{C_1}V_0}=\left(\frac{Z_{R_2}Z_{C_2}+Z_{C_1}Z^2C_2+Z_{C_1}Z_{C_2}R_2+Z_{R_2}Z_{C_2}Z_{R_1}+Z_{C_1}Z_{C_2}Z_{R_1}+Z_{C_1}Z_{R_2}Z_{R_1}-Z_{R_1}Z_{R_2}Z_{C_1}-KZ_{R_1}Z_{C_1}Z_{C_2}}{KZ_{R_1}Z_{R_2}Z_{C_1}Z_{C_2}}\right)$$

$$\frac{V_1}{V_0}=\frac{\cancel{Z_{C_1}}(Z_{R_2}Z^2C_2+Z_{C_1}Z^2C_2+Z_{C_1}Z_{C_2}R_2+Z_{R_2}Z_{C_2}Z_{R_1}+Z_{C_1}Z_{C_2}Z_{R_1}-KZ_{R_1}Z_{C_1}Z_{C_2})}{KZ_{R_1}Z_{R_2}\cancel{Z_{C_1}}Z_{C_2}}$$

$$\frac{V_0}{V_1}=\frac{KZ_{R_1}Z_{R_2}Z_{C_2}}{Z_{R_2}Z^2C_2+Z_{C_1}Z^2C_2+Z_{C_1}Z_{C_2}R_2+Z_{R_2}Z_{C_2}Z_{R_1}+Z_{C_1}Z_{C_2}Z_{R_1}-KZ_{R_1}Z_{C_1}Z_{C_2}}$$

$$\frac{V_0}{V_1}=H(S)=\frac{\frac{KR_1R_2}{SC_2}}{\frac{R_2}{S^2C_2^2}+\frac{1}{S^3C_2^2C_1}+\frac{R_2}{S^2C_1C_2}+\frac{R_2R_1}{SC_2}+\frac{R_1}{S^2C_1C_2}-\frac{KR_1}{S^2C_1C_2}}$$

$$\frac{\frac{KR_1R_2}{SC_2}}{\frac{SR_2C_1+1+S^2R_1R_2C_1C_2+SC_2R_1-KR_1SC_2+SR_2C_2}{S^3C_1C_2^2}}$$

High Pass Filter with gain Derivation Workout part 1

$$H(s) = \frac{s^3 K R_1 R_2 C_1 C_2^2}{s C_2 \left(s R_2 C_1 + C_2 + s^2 R_1 R_2 C_1 C_2 + s C_2 R_1 - K R_1 s C_2 + s R_2 C_2 \right)}$$

$$H(s) = \frac{s^2 K R_1 R_2 C_1 C_2}{s^2 R_1 R_2 C_1 C_2 + s(R_2 C_1 + C_2 R_1 - K R_1 C_2 + R_2 C_2) + 1}$$

$$H(s) = \frac{s^2 K R_1 R_2 C_1 C_2}{s^2 R_1 R_2 C_1 C_2 + s(R_2 C_1 + (1-K) R_1 C_2 + R_2 C_2) + 1}$$

$\Downarrow$

OR

$\Downarrow$

$$H(s) = \frac{s^2 K R_1 R_2 C_1 C_2}{s^2 R_1 R_2 C_1 C_2 + s\left(R_2 C_1 + R_2 C_2 - \frac{R_4}{R_3} R_1 C_2\right) + 1}$$

$\Downarrow$

or

$\Downarrow$

$$H(s) = \frac{K s^2}{s^2 + s\left(\frac{1}{R_1 C_2} + \frac{1}{R_1 C_1} - \frac{R_4}{R_2 R_3 C_1}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

High Pass Filter with gain Derivation Workout part 2

```
%% transfer function 2nd HPF
%% Components
R1=1910;
R2=26200;
R3=1000;
R4=22000;
C1=1.5*10^-9;
C2=C1;
%% K = GAIN
K=(R4/R3)+1;
%% Q Factor
Q_top=sqrt(R2*R1*C1*C2);
Q_bot=(R2*C2)+(R2*C1)+R1*C2*(1-K);
Q=Q_top/Q_bot
%% Cut-off frequency
fc=1/(2*pi*sqrt(R1*R2*C1*C2))
%% Transfer Function
num=[K,0,0];
den=[1,(1/(R1*C2))+(1/(R1*C1))-(R4/(R2*R3*C1)),(1/(R1*R2*C1*C2))];
H=tf(num,den)
%% Plot
options=bodeoptions;
options.FreqUnits = 'Hz';
figure(1)
bode(H,options);
grid
```

High Pass Filter with gain in MATLAB

MATLAB codes for Butterworth,Chebyshev and Bessel

```matlab
%% TASK 1 %%
%% Butterworth %%
for n=1:2:10 %% condition for the number of orders, from 1 to 10, intervals of 2
Wn = 0.4; %%cut-off frequency
[num,den]=butter(n,Wn,'high'); %% Butterworth command for Low-Pass filter
sys=tf(num,den) %% Transfer Function
[h,w]=freqz(num,den); %%
%% Ideal Filter
f = [0 0.4 0.4 1]; %% normalized parameters of the ideal Filter
m = [0 0 1 1];%% Magnitude of the ideal Filter
b=fir2(n,f,m)
[h1,w1]=freqz(b,1);
%% Graph plotted
figure (1)
plot(w/pi,abs(h),'LineWidth',2) %% Butterworth Filter plot
xlabel('Normalized frequency (rad/sample)','FontSize',16,'FontWeight','bold')%%Label X
ylabel('Amplitude','FontSize',16,'FontWeight','bold') %%Label Y
legend('1st Order','3rd Order','5th Order','7th Order','9th Order','Ideal') %% Legend
hold on %% command to display more than one plot
end
grid
plot(f,m,'LineWidth',2) %% Ideal Filter plot
hold off
%% TASK 1 %%
%% Chebyshev %%
for n=2:2:8 %% condition for the number of orders, from 1 to 10, intervals of 2
dB = 1; %% Peak to Peak
Wn = 0.7; %%cut-off frequency
[num,den]=cheby1(n,dB,Wn,'low'); %% Cheby1 or cheby2 command for Low-Pass filter
sys=tf(num,den) %% Transfer Function
[h,w]=freqz(num,den); %%
%% PLOT %%
figure (2)
grid on
plot(w,abs(h),'LineWidth',3)
xlim ([0 3])
ylim ([0 1.1])
ylabel('Amplitude','FontSize',18,'FontWeight','bold')
xlabel('Normalized Frequency (rad/sample)','FontSize',18,'FontWeight','bold')
legend('2nd Order', '4th Order','6th Order','8th order','FontSize',16,'FontWeight','bold')
title('Chebyshev','FontSize',20,'FontWeight','bold')
hold on
end
%% Bessel
for n=2:2:8 %% Number of orders
[b,a] = besself(n,10000); %% Bessel command
freqs(b,a)
grid on

ylim ([0 1.1])
ylabel('Amplitude','FontSize',18,'FontWeight','bold')
xlabel('Frequency (rad/s)','FontSize',18,'FontWeight','bold')
legend('2nd Order', '4th Order','6th Order','8th order','FontSize',16,'FontWeight','bold')
title('Bessel','FontSize',20,'FontWeight','bold')
hold on
end
hold off
```